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A dynamic nonlinear model based on economic adjustment coefficients for optimizing floating share in gold mines under price volatility

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ABSTRACT

This study presents a dynamic nonlinear model for optimizing the floating-share in gold mining operations under price volatility. The model integrates three economic adjustment coefficients—base coefficient (a), adjustment power (p), and partial correction coefficient (d)—which collectively enable automatic responses and dynamic adaptation to fluctuations in gold prices. Sensitivity analysis reveals that the parameter (p) is the principal determinant of the curvature and growth rate of the floating-share function, whereas the parameters (a) and (d) predominantly influence the initial slope and baseline position of the function. Regarding the amount of the share premium in Iranian gold mining contracts, it is a fixed coefficient, and similarly, in the world, share premiums either have a fixed amount and or increase in a stepwise manner. Therefore, the presented nonlinear model was validated using monthly data on the price of 24-karat gold over the past 42 months. Then, seven scenarios were defined to determine the value of the floating-share based on the needs assessment and mining contracts. then, key indicators including mean floating-share price, standard deviation, risk, conservative profit of the mine owner, coefficient of variation (CV), profit percentage relative to the best scenario, and the composite decision-making index (CI)— were used to evaluate and compare the results of the seven scenarios. The results showed that scenario number 4, which aims to start with a floating-share price that is always at least 10 % higher than the minimum suggested share price and, in 10 % of cases, gives people who earn their final share price, was selected as the optimal scenario with the highest profitability and the lowest average risk. Overall, the proposed model provides a quantitative, adaptive, and efficient framework for financial decision-making, joint venture contract design, and risk assessment in gold mining operations. Its implementation can support more informed and resilient strategic decisions in the face of market volatility.

KEYWORDS

Dynamic model, floating-share, gold price volatility, economic adjustment coefficients, financial sustainability

I. INTRODUCTION

Mining activities, particularly gold mining, are inherently high-risk and yield volatile returns. The distribution of benefits between the operator and the resource owner (government or private investor) requires mechanisms that are flexible, transparent, and equitable. One commonly used instrument to achieve this objective is the floating royalty, which allocates a percentage of production or sales revenue to the resource owner or government. This mechanism can be structured as a fixed rate, revenue-dependent, or tiered system (Otto et al.,2006). Two primary approaches exist for designing floating royalties. Linear approaches are simple and transparent but offer limited flexibility. In contrast, tiered or market-dependent approaches provide greater flexibility in risk-sharing, although they involve more regulatory complexity and may distort economic behavior (Conrad, 2018; Akinseye et al., 2021). The high volatility of gold prices, driven by monetary policies, global fluctuations, and institutional investment flows, underscores the importance of dynamic adjustment functions (Otto et al.,2006, Conrad, 2018, Akinseye et al., 2021, Ranjan, 2021). Despite numerous studies, a significant gap remains in dynamic and nonlinear modeling using power adjustment functions with partial correction coefficients, and few studies have evaluated their performance under extreme market conditions (Ranjan, 2021). The aim of this study is to propose a dynamic nonlinear model for optimizing floating royalties, providing a framework that, through adjustment coefficients and sensitivity analysis, ensures minimum government revenue during price downturns while approaching the maximum share during periods of high prices. Secondary objectives include estimating the optimal parameters (power, economic adjustment coefficient, and partial correction coefficient) and comparing the model's performance with the traditional linear method in terms of revenue stability and

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sensitivity. The structure of the paper is as follows: Section 2 presents a review of the literature and international experiences; Section 3 outlines the overall framework of the model; Section 4 details the analysis of various scenarios; and Section 5 provides the conclusions.

II. Literature Review and Theoretical Background

A royalty is a payment made by the operating company to the resource owner (government or local authority) in exchange for extraction rights (IMF, 2012). In mining finance literature, royalty schemes are generally classified into three categories: fixed-rate, production- or sales-value-based, and variable (sliding) royalties, which are functions of price, profit, or production volume (IGF, 2024). Studies indicate that simple schemes, such as fixed rates, offer limited flexibility in responding to price and cost fluctuations (Benninger et al., 2024). In such systems, government revenue does not increase during price upswings and declines during price downturns. For instance, the OECD report (2024) indicates that a fixed 3% royalty on gold resulted in a revenue loss exceeding USD 100 million during periods of price volatility. Raising the fixed rate without considering economic conditions may also discourage company investments and promote risk-averse behavior (Laniran et al., 2024). Consequently, many countries have replaced fixed rates with tiered or sliding royalty systems to provide greater flexibility and enhance government revenue. The linear model is typically defined as follows (Birch, 2016):

$$R = R_{min} + \alpha.(P - P_0) \quad (1)$$

where R is the final government royalty rate, $R_{\min}$ is the minimum base rate, P is the gold price, P_0 is the reference price, and α is the rate adjustment coefficient. This relationship has been widely applied in countries such as Ghana, Mali, Chile, and Australia (IMF, 2023). Recent studies suggest instead of linear rates, nonlinear adjustment functions (F) should be employed to enhance system responsiveness. The general form of this function is expressed as follows:

$$G = F_0 + a.F^b \quad (2)$$

Here, G represents the adjusted share, F denotes the performance index or production coefficient, F_0 is the base share, a is the sensitivity coefficient, and b is the nonlinear exponent. As reported by Bertinelli et al. (2019), these models demonstrate higher efficiency under conditions of price volatility (Bertinelli et al., 2019).

A review of international experiences indicates that countries typically employ a combination of these models for gold extraction. These models are classified from simple to complex and serve as the basis for developing the floating and dynamic share models in this study. In particular, Model No. 5 in Table 1 has been used as the primary foundation for the analysis and development of the floating-share framework in the present research. Despite extensive research in mining financial policy, three fundamental gaps remain:

- The lack of dynamic nonlinear models for determining floating-shares in gold mines.
- The limited alignment between theoretical models and actual price data.
- Performance indicators and the scarcity of comparative studies between traditional and nonlinear methods.

The present study aims to address these gaps by presenting a framework based on the proposed model for nonlinear and adjustable floating-shares.

Table 1. Review of international experiences

Model	Title	Mathematical Formula for Share (R)	Countries of Application	References
1	Flat royalty	$R = R_0$	South Africa, Iran	(IMF, 2012)
2	Production-Based Rate	$R = \alpha.Q$	Bolivia	(Hogan, 2008)
3	Value-Based Rate	$R = \alpha.P \times Q$	Ghana, Tanzania, Canada, Australia	(IMF, 2023, Kwon et al., 2020)
4	Profit-Based Rate	$R = \alpha.(P - C)$	Chile, Canada, Peru	(Bertinelli et al., 2019)
5	Sliding scale	$Y = Y_{min} + \frac{r - r_{min}}{r_{max} - r_{min}}(Y_{max} - Y_{min})$	Mali, Ghana, South Africa	(Birch, 2016)
6	Production Sharing-PSC	R=β.((Q.P-C))/ (Q.P))	Indonesia	(NRGI ,2024)
7	Resource Rent Tax Model-RRT	$R = \tau.(NP - r.K)$	Australia, Norway	(Bertinelli et al., 2019)
8	Hybrid Royalty + Profit Share + Price Adjustment	$R = \alpha.P.Q + \beta.(TR - C) + \gamma.\frac{P - P_0}{P_0}.(Q.P)$	Peru, Philippines	(IGF, 2022)

Table 2. Parameters used in Table 1

Model Number	Parameter	Description
1	R_0	Fixed rate per unit of production
2	α	Share Rate
	Q	Metal Sales Volume
3	α	Share Rate (typically 3–5%)
	P	Price per Unit of Metal
	Q	Metal Sales Volume
4	α	Share Rate (typically 15–30%)
	P	Total Revenue
	C	Production Costs
5	Y_{min}	Minimum Share
	Y_{max}	Maximum Share
	R_{min} & r_{max}	Minimum and Maximum Mine Profitability
6	β	Government Participation Coefficient (typically between 0.5 and 0.8)
	Q	Production Quantity
	P	Selling Price
	C	Total Cost (Capital + Operating Costs)
7	τ	Tax Rate on Excess Profit (typically 30%–50%)
	NP	Net Project Profit
	K	Cumulative Investment
	r	Required Rate of Return
8	γ	Price Adjustment Coefficient
	P_0	Base Gold Price
	TR	Total Revenue

III. Overall Framework of the Proposed Model

Traditional methods for allocating floating-shares are largely based on contractual agreements and practical experience in mining operations, and they lack a solid theoretical foundation and coherent quantitative modeling. These approaches are typically linear and static, highly sensitive to price fluctuations, and incapable of simultaneously controlling both minimum and maximum share levels. In addition, their applicability in financial risk analysis and the design of participatory contracts is limited.

Classical models generally assume a simple linear relationship between the royalty share and the adjustment coefficient, whereas empirical evidence indicates that this relationship is, in reality, nonlinear and dynamic. Accordingly, the objective of this study is to develop a nonlinear, dynamic, and adjustable framework for determining floating-shares in gold mines—one that, under volatile price conditions, preserves a guaranteed minimum share while enabling the attainment of an optimal and maximum share.

In this model, the floating-share is defined as a function of the performance coefficient (F) within the interval [0,1]. This coefficient reflects the relative state of the market and incorporates a combination of economic indicators, such as the global gold price, extraction costs, and project returns. To enhance flexibility and accuracy, a non-linear power-form function is proposed for calculating the final share, which constitutes the core of the proposed model. By gradually adjusting the share in response to economic fluctuations, the model provides an effective tool for optimizing financial and contractual decision-making in the mining sector. The proposed model is presented as follows.

$$R = R_{min} + (R_{max} - R_{min}) \times \min((a.F + d)^p, 1)$$
$$F = \frac{P_{ave} - P_{min}}{P_{max} - P_{min}} \tag{4}$$

Where (R) represents the floating-share, Rmin is the minimum share, and Rmax is the maximum share. The function min (0,1) acts as a limiting operator, ensuring that the share does not exceed the predetermined maximum value. (F) is the performance coefficient or economic index, ranging from 0 to 1. The parameter (d) is a partial adjustment coefficient that prevents excessive reduction of the share at low (F) values. The exponent (p) determines the intensity of the nonlinear response of the floating-share to (F), while the coefficient (a) acts as a scaling factor that governs the impact of (F) on the final share. Pave, Pmin, and Pmax represent the average, minimum, and maximum prices per gram of 24- karat gold, respectively, during the calculation period (monthly or quarterly).

A. Analytical Solution of Unknown Parameters in Floating-share Calculation

To analytically solve Eq. (3), it is necessary to determine the three unknown parameters (a), (d), and (p). Two distinct scenarios are considered for estimating these parameters, in which, for each scenario, the solution method, the required supplementary equations, and the proposed numerical approach are described.

In the first case, if $(a.F + d)^p < 1$, the model exhibits elastic behavior, and it is assumed that the numerical value of $(a.F_{tmin} + d)^p$ equals ε. Accordingly, the partial adjustment coefficient (d) is defined as follows:

$$(a.F_{tmin} + d)^p = \varepsilon \rightarrow d = \varepsilon^{1/p} - a.F_{tmin} \tag{5}$$

In the second case, if$(\mathrm{a.F} + \mathrm{d})^{\mathrm{p}} \geq 1$, the model reaches a saturation state, and it is assumed that the numerical value of $(\mathrm{a.F_{tmax}} + \mathrm{d})^{\mathrm{p}}$ equals one.

By substituting the parameter (d) from Eq. (4) into$(\mathrm{a.F_{tmax}} + \mathrm{d})^{\mathrm{p}}$, the economic adjustment coefficient a is defined as follows:

$$(a.F_{tmax} + (\varepsilon^{1/p} - a.F_{tmin}))^p = 1 \Rightarrow a.(F_{tmax} - F_{tmin}) + \varepsilon^{1/p} = 1 \Rightarrow a = \frac{1 - \varepsilon^{1/p}}{F_{tmax} - F_{tmin}} \tag{6}$$

F_{tmax} and F_{tmin} are target functions, which can be expressed as a percentage of the performance coefficient F. If minimum and maximum conditions are defined in the problem assumptions and it is desired that the share results fluctuate within a specific range, Eq.s (7) and (8) are used. Furthermore, if no limiting constraints are imposed for the problem solution, the range between the minimum and maximum values of F will be utilized.

$$F_{tmax} = F_{min} + n(1\text{-}m_1)\left(\frac{F_{max} - F_{min}}{n-1}\right) \tag{7}$$

$$F_{tmin} = F_{min} + n \times \mathrm{m}_2 \times \left(\frac{F_{max} - F_{min}}{n-1}\right) \tag{8}$$

Where, (n) represents the number of (F) points, while F_{max} and F_{min} denote the maximum and minimum values of the performance coefficient (F), respectively. Additionally, m_1 % of the data should reach the maximum final share, and m_2 % of the data should remain above the minimum final share .To ensure the model's validity, the partial adjustment coefficient (d) must be positive; therefore, we have:

$$d = \varepsilon^{1/p} - a.F_{tmin} > 0 \Rightarrow a.F_{tmin} < \varepsilon^{1/p} \tag{9}$$

By substituting Eq. (5) into Eq. (8), we obtain:

$$\frac{F_{tmin}}{F_{tmax} - F_{tmin}} < \frac{\varepsilon^{1/p}}{1 - \varepsilon^{1/p}}$$

If both denominators are positive ($F_{tmax} > F_{tmin}$ and $1 - \varepsilon^{1/p} > 0$), the two fractions can be multiplied together to yield the following relationship:

$$\frac{F_{tmin}}{F_{tmax}} < \varepsilon^{1/p}$$

The ratio between F_{tmin} and $F_{tmax}$is denoted by r:

$$r = \frac{F_{tmin}}{F_{tmax}} \ or \ \frac{F_{min}}{F_{max}} \tag{10}$$

This ratio represents the compactness of the active region of the function. The smaller the r value, the greater the distance between $F_{tmin}$and F_{tmax}, resulting in a steeper growth of the function. In the absence of limiting conditions for defining the function (R), the ratio between the minimum and maximum (F) values is used. Therefore, the minimum value of the exponent (p) is given by:

$$r < \varepsilon^{1/p} \Rightarrow Ln(r) < Ln(\varepsilon^{1/p}) \Rightarrow p > \frac{Ln(\varepsilon)}{Ln(r)} \Rightarrow p_{min} = \frac{Ln(\varepsilon)}{Ln(r)} \tag{11}$$

B. Sensitivity Analysis of Parameters Influencing the Calculation of Floating-share

1) *Sensitivity Analysis of the Floating-share with Respect to Parameter (p)*

The sensitivity analysis of the floating-share (R) with respect to the exponent (p) reveals nonlinear behavior near the saturation limit. The proposed model $R(F, p)$ exhibits an elastic–saturation structure. In the initial phase, an increase in (p) leads to rapid and pronounced growth of (R). However, once the saturation point is reached, further increases in the exponent have a negligible effect on the output. This behavior indicates that precise adjustment of the exponent (p) can significantly control the dynamics of the participation rate and the stability of the project's financial system. The objective of this section is to examine how variations in the exponent (p) alter the response rate of the function (R) to the performance coefficient (F), and to identify the point at which this sensitivity reaches its maximum and the system approaches saturation. In fact, differentiation is employed to uncover the elastic behavior of the function (when (R) is highly dependent on (F)) and to identify the saturation threshold (when further increases in the exponent no longer have an effect). The derivative of the function with respect to the variable (p) (marginal sensitivity) indicates how much (R) changes for a small increase in (F), assuming that the exponent (p) remains constant. Next, Eq. (3) is differentiated with respect to (F). Accordingly, we obtain:

$$\frac{\partial R}{\partial F} = (R_{max} - R_{min})ap(aF + d)^{p-1} \tag{12}$$

We also take the derivative regarding (p) to examine the direct effect of power variations in the floating–share exponent function. This derivative indicates the extent to which variations in the power parameter (p) can alter the growth rate (R). This component represents the core of the sensitivity analysis.

$$\frac{\partial R}{\partial p} = (a.F + d)^p Ln(a.F + d) \tag{13}$$

When both (F) and (p) vary simultaneously, the cross-partial derivative is employed. The resulting sign determines whether an increase in the exponent amplifies or weakens the sensitivity of (R) to changes in the performance measure (F).

$$\frac{\partial}{\partial p}\left(\frac{\partial R}{\partial F}\right) = a(R_{max} - R_{min})(aF + d)^{p-1} + \left(1 + pLn(a.F + d)\right) \tag{14}$$

To identify the Elastic Threshold, we set expression $(1 + pLn(a.F + d))$ equal to zero; therefore:

$$p_{op} = \frac{-1}{Ln(a.F_{tmin} + d)} \tag{15}$$

In fact, the point obtained from the second derivative represents the inflection point of (R) with respect to (F), indicating where the effect of F on (R) shifts from accelerating to decelerating growth, or vice versa.

2) *Sensitivity Analysis of the Floating-Share with Respect to the Parameter* (a)

The sensitivity analysis of the floating-share (R) with respect to parameter a reveals a significant nonlinear dependency. Next, the first derivative of R with respect to a is examined. Therefore:

$$\frac{\partial R}{\partial a} = pF(R_{max} - R_{min})(aF + d)^{p-1} \tag{16}$$

The above relations indicate that the response of (R) to variations in a gradually increases as (a) rises ($p - 1 > 0$). For small values of a, the derivative $\frac{\partial R}{\partial a}$ is relatively low, and the effect of variations in (a) is limited. As (a) approaches the threshold $(aF + d)^p = 1$, the sensitivity reaches its maximum, indicating a region of high impact. Beyond this point, (R) reaches its maximum value R_{max}, and further variations in (a). This nonlinear behavior highlights the importance of selecting an optimal parameter a. A moderate increase in a can lead to a significant rise in R, whereas excessive increases result in diminishing returns. Therefore, the formula for the first derivative $\frac{\partial R}{\partial a}$ provides (a) quantitative and analytical basis for determining the optimal value of a and maximizing the floating-share before reaching the saturation threshold. To determine the value of a that leads to saturation of (R), the following procedure is applied.

$$(aF + d)^p = 1 \Rightarrow a_{max} = \frac{1 - d}{F_{tmax}} \tag{17}$$

An analysis of the second derivative of the floating-share (R) with respect to parameter (a) enables the assessment of the nonlinearity of sensitivity and the identification of optimal points. The second derivative in the unsaturated state is given as follows:

$$\frac{\partial^2 R}{\partial^2 a} = (R_{max} - R_{min})p(p - 1)F^2(aF + d)^{p-2} \tag{18}$$

Given the values of parameters (p) and (F), the second derivative is positive, indicating that the function (R) is convex with respect to (a) .This means that the sensitivity of (R) to variations in (a) increases with (a) until the saturation threshold is reached. After reaching this point, the effect of increasing (a) on (R) ceases, and the second derivative is no longer applicable. This nonlinear behavior indicates that the maximum effect of parameter (a) on (R) occurs approximately near the saturation point. Therefore, determining the value of (a) just bellow the saturation threshold is essential for maximizing its effect. The analysis of the second derivative, alongside the first derivative, provides a quantitative and analytical framework for optimizing parameter (a) which can be applied in the design of regulatory policies or for economic decision-making related to the floating-share.

3) *Sensitivity Analysis of the Floating-share Magnitude with Respect to Parameter (d)*

The sensitivity analysis of the floating-share magnitude (R) with respect to parameter (d) indicates that this parameter plays a pivotal role in determining the baseline position and the response condition of (R) to variations in (F) and (a). Accordingly, the first derivative of (R) with respect to (d) in the unsaturated state is expressed as follows.

$$\frac{\partial R}{\partial d} = p(R_{max} - R_{min})(aF + d)^{p-1} \tag{19}$$

The results indicate that the first derivative reveals a direct and nonlinear dependency of (R) on (d), such that increasing (d) leads to a more rapid growth of (R) as it approaches the saturation limit. The second derivative is positive and confirms the convexity of the function with respect to (d), implying that the sensitivity of (R) to variations in (d) increases with larger values of (d).

$$\frac{\partial^2 R}{\partial^2 d} = p.(p - 1)(R_{max} - R_{min})(aF + d)^{p-2} \tag{20}$$

This analysis demonstrates that selecting an optimal value for (d) can serve as an effective tool for enhancing the function's response and increasing the influence of variations in other parameters, without causing the function to reach its saturation limit prematurely. In other words, parameter (d) acts as a fundamental controller for both the slope and the sensitivity threshold of (R), and determining its optimal value is crucial for policy design and economic decision-making related to the floating-share magnitude. Accordingly, in this study, the sensitivity of the function (R) with respect to the three key parameters (p), (a), and (d) was systematically analyzed. The findings indicate that parameter (p) is the most influential variable in determining the nature and intensity of the function's response to changes in (F). By governing the curvature of the curve and the function's growth rate, this parameter plays the primary role in shaping the behavior of the floating-share magnitude. The practical optimal value of (p) is derived according to Eq. (14) to ensure model stability and to maintain control over the increasing trend of (R).

In contrast, parameters (a) and (d) exert more uniform and less structural effects on the function. Specifically, parameter (a) mainly determines the initial slope of the relationship with (F), while parameter (d) merely causes a limited shift in the initial value. Since neither parameter is capable of altering the fundamental curvature of the function, their roles are essentially confined to adjusting the scale and initial position of the function. Thus, it can be concluded that (p), as the principal shaping factor of the floating-share magnitude function, possesses a greater structural significance compared to the other two parameters.

IV. Analysis of Different Contractual Scenarios and Their Impact on the Floating-share Received by the Mine Owner

Seven potential scenarios are considered within gold mine contractual models, as follows:

1. The received floating-share is linearly distributed between the proposed minimum and maximum shares (Model 5 of Table 1). This model is designed based on a sliding scale and is used in countries such as Mali, Ghana, and South Africa.
2. The received floating-share is distributed nonlinearly between the proposed minimum and maximum shares (the model proposed by the author).

The remaining scenarios are defined based on the mine management request and possible states.

3. The initial percentage of the received floating-share is always at least 5% higher than the proposed minimum, reaching its maximum final share in 10% of possible cases.
4. The initial percentage of the received floating-share is always at least 10% higher than the proposed minimum, reaching its maximum final share in 10% of possible cases.
5. The initial percentage of the received floating-share is always at least 5% higher than the proposed minimum, reaching its maximum final share in 20% of possible cases.
6. The initial percentage of the received floating-share is always at least 5% higher than the proposed minimum, reaching its maximum final share in 30% of possible cases.
7. The initial percentage of the received floating-share is always at least 5% higher than the proposed minimum, reaching its maximum final share in 40% of possible cases.

To analyze and determine the unknowns in Eq. (3), that are applied within these contractual scenarios, the parameter (F) is first calculated. To evaluate the performance of each scenario, the monthly 24-carat gold price data over a 42-month period were collected and analyzed. Using Eq. (3-b), (F) was computed for each month and is presented in Table 3. The parameter (F) varies between 0.231 and 0.764. These calculated values serve as the basis for calculating the floating-share for each scenario, enabling a quantitative and comparative assessment of how different contractual structures affect the mine owner's economic returns. The computed (F) values are illustrated in Fig. 1.

As an example, Scenario 7 is analyzed both quantitatively and qualitatively based on the following assumptions. Its outcomes are compared with those of Model 5 in Table 1, providing a precise evaluation of the potential effects of different contractual structures on the mine owner's floating-share.

Assumptions for the problem:

- The minimum and maximum shares are assumed to be 25% and 30%, respectively.
- Variations of the performance coefficient (F) over the 42-month period lie within the range $F \in [0.231, 0.746]$.
- Condition 1: The floating-share is always considered at least 5% higher than the minimum share (25%).
- Condition 2: The floating-share reaches the maximum share (30%) in 40% of the cases.
- Parameters (a), (d), and (p) are all greater than zero.

Assumptions for the problem:

- The minimum and maximum shares are assumed to be 25% and 30%, respectively.

Table 3. Gold prices and performance coefficient (F)

Date	P_{min}	P_{ave}	P_{max}	F	Date	P_{min}	P_{ave}	P_{max}	F
20/04/2022	16,478,000	17,163,455	17,656,000	0.5819	20/01/2024	32,740,000	34,371,880	36,390,000	0.447
21/05/2022	17,038,000	17,642,182	18,601,000	0.387	19/02/2024	36,144,000	37,210,048	39,379,000	0.330
21/06/2022	18,377,000	19,030,960	20,110,000	0.377	20/03/2024	37,320,000	40,281,000	43,381,000	0.489
22/07/2022	17,992,000	18,544,750	19,429,000	0.385	20/04/2024	42,968,000	47,172,350	51,720,000	0.480
22/08/2022	15,957,000	17,798,375	18,368,000	0.764	20/05/2024	44,076,000	46,072,423	49,500,000	0.368
22/09/2022	16,647,000	17,166,192	17,838,000	0.436	20/06/2024	43,704,000	44,571,773	45,286,000	0.549
22/10/2022	17,478,000	18,052,227	18,648,000	0.491	20/07/2024	44,544,000	45,683,957	47,309,000	0.412
21/11/2022	17,826,000	19,126,615	20,273,000	0.532	20/08/2024	44,581,000	46,654,037	49,300,000	0.439
21/12/2022	19,940,000	21,523,240	23,591,000	0.434	20/09/2024	47,770,000	48,681,048	49,842,000	0.440
20/01/2023	23,936,000	25,214,583	27,113,000	0.402	20/10/2024	50,738,000	54,142,000	57,673,000	0.491
19/02/2023	27,153,000	28,214,652	30,403,000	0/327	19/11/2024	56,617,000	59,865,308	63,343,000	0.483
20/03/2023	26,568,000	31,310,789	35,208,000	0.549	19/12/2024	59,603,000	62,571,917	66,258,000	0.446
20/04/2023	33,158,000	34,078,333	34,740,000	0.582	19/01/2025	65,224,000	69,064,400	71,380,000	0.624
20/05/2023	33,331,000	35,083,435	36,640,000	0.530	18/02/2025	70,395,000	79,244,333	88,267,000	0.495
21/06/2023	29,963,000	32,201,480	34,365,000	0.509	20/03/2025	85,032,000	90,369,375	108,124,000	0.231
21/07/2023	30,545,000	31,366,720	32,395,000	0.444	20/04/2025	87,390,000	99,528,600	110,417,000	0.527
22/08/2023	30,175,000	31,023,750	31,890,000	0.495	20/05/2025	82,942,000	87,511,269	90,757,000	0.585
21/09/2023	30,674,000	31,164,870	31,610,000	0.524	20/06/2025	85,546,000	86,936,235	90,320,000	0.291
22/10/2023	29,865,000	31,250,071	32,712,000	0.487	20/07/2025	89,584,000	94,681,381	99,995,000	0.490
21/11/2023	32,041,000	32,667,154	33,715,000	0.374	25/08/2025	93,447,000	99,101,458	104,603,000	0.507
21/12/2023	32,192,000	32,836,087	33,672,000	0.435	21/09/2025	103,528,000	115,263,130	128,988,000	0.461

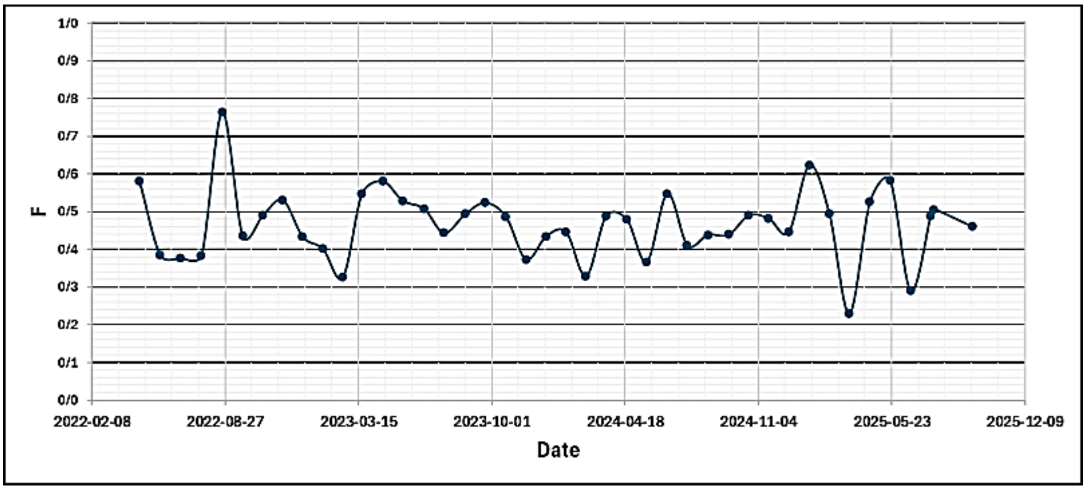

Fig. 1. Values of (F) over the 42-month period

Therefore, to ensure compliance with Condition 1, it is assumed that the floating-share is always at least 5% higher than the minimum defined share (25%). Accordingly, Accordingly, the lower bound of the floating-share in the calculations is set at 26.25%.

$$R = (1 + 0.05) \times R_{min} = 26.25$$

By substituting the value of 26.25% into Eq. (3-a), we obtain:

$$min((a.F + d)^p, 1) = \frac{R - R_{min}}{R_{max} - R_{min}} = \frac{26.25 - 25}{30 - 25} \to \varepsilon = 0.25 < 1$$

To satisfy Condition 2, which states that in 40% of cases the floating-share should reach the assumed maximum share (30%), the expression within the minimum function in Eq. (3-a) must equal one, so that the model output attains the ceiling value (maximum share) within that interval. Subsequently, using Eq. (6), the parameter (F_t) is calculated for both of the aforementioned conditions:

$$F_{tmax} = 0.231 + 42 \times (1 - 0.40) \times 0.0125 = 0.545$$

$$F_{tmin} = 0.231 + 42 \times 0.05 \times 0.0125 = 0.25725$$

The necessary condition for (d > 0) is as follows:

$$r = \frac{F_{tmin}}{F_{tmax}} = \frac{0.25725}{0.545} = 0.47115$$

$$p_{min} = \frac{Ln(0.25)}{Ln(0.47115)} = 1.85$$

To maintain a safe margin, (p) is chosen slightly higher than p_{min}.

$$p = 0.1 + p_{min} = 1.95$$

By substituting the value of (p) into Eq.s (4) and (5), we obtain:

$$a = \frac{1 - \varepsilon^{1/p}}{F_{tmax} - F_{tmin}} = \frac{1 - 0.25^{1/95}}{0.545 - 0.25725} = 1.762$$

$$d = \varepsilon^{1/p} - a.F_{tmin} = 0.4912 - 1.762 \times 0.25725 = 0.0379$$

The parametric analysis of the proposed model indicates that the optimal combination of coefficients—including a power of 1.95, an economic adjustment factor of 1.762, and a partial correction coefficient of 0.0379—yields the highest floating-share values compared to other methods. Using these optimal values, Eq. (3) can be rewritten as follows:

$$R = R_{min} + (R_{max} - R_{min}) \times \min((1.762F + 0.0379)^{1.95}, 1) \quad (21)$$

Based on Eq. (21) and (F) values ranging from 0.231 to 0.764, the floating-share was calculated using both the traditional and proposed methods and is presented in Table 4.

Based on Table 4 and Fig. 2, the following observations can be made:

As (F) increases from 0.231 to 0.764, the proposed model value rises from 26.03% to 30%. The overall trend is upward; however, after reaching approximately 30%, the model reaches a saturation level, and further increases in (F) have no significant effect.

In cases where (F≥F-t), the received floating-share is below the maximum, representing the natural fluctuations of the floating-share. This portion reflects the sensitivity of the floating-share to performance variations.

For (F≥F-t), the proposed model remains nearly constant at the maximum floating-share. This value represents the upper bound of the model and the legal or practical ceiling of the floating-share, corresponding to high-performance data where the floating-share has reached its maximum allowable or predicted value.

The proposed model effectively captures the effect of (F). At the lower end of the (F) range, a significant impact is observed, but the model then reaches a saturation limit. This behavior indicates a balance between sensitivity and model stability and confirms that selecting optimal values of (F) can maximize the floating-share.

Accordingly, the proposed model is capable of effectively simulating the adaptive and dynamic behavior of the floating-share in response to price fluctuations, providing a reliable basis for economic and contractual decision-making in gold mines. In Fig. 2, the results obtained from the traditional methods and the proposed model are presented comparatively.

Based on Eq. (21), the effect of variations in the power parameter on the floating-share was examined over the range of 0.5 to 3, and the corresponding trend is presented in Fig. 3. As observed, with an increase in the power parameter, the final floating-share decreases within the elastic region. According to the calculations and analyses conducted, the optimal power is 1.95.

The effect of parameter (a) on the floating-share was examined over the range of 0.6 to 2.5, and the corresponding trend is shown in Fig. 4. This nonlinear behavior indicates that the maximum impact of parameter (a) on (R) occurs near the saturation point. Based on the calculations and analyses conducted, the optimal value of (a) is 1.762.

Table 4. Comparison of floating-share results between traditional and proposed methods

Number	F	Method		Number	F	Method	
		Traditional	Proposed			Traditional	Proposed
1	0.231	26.16	26.03	22	0.480	27.40	28.97
2	0.297	26.46	26.56	23	0.483	27.42	29.03
3	0.327	26.64	26.93	24	0.487	27.44	29.06
4	0.330	26.65	26.96	25	0.489	27.45	29.08
5	0.368	26.84	27.40	26	0.490	27.45	29.10
6	0.374	26.87	27.47	27	0.491	27.46	29.10
7	0.377	26.89	27.51	28	0.494	27.47	29.16
8	0.385	26.93	27.60	29	0.495	27.48	29.35
9	0.387	26.94	27.63	30	0.507	27.54	29.38
10	0.402	27.01	27.83	31	0.509	27.55	28.64
11	0.412	27.03	27.96	32	0.524	27.62	29.68
12	0.427	27.13	28.25	33	0.527	27.64	29.72
13	0.434	27.17	28.27	34	0.530	27.65	29.75
14	0.435	27.18	28.28	35	0.532	27.66	30
15	0.436	27.18	28.33	36	0.548	27.75	30
16	0.439	27.20	28.34	37	0.549	27.75	30
17	0.440	27.20	28.40	38	0.581	27.91	30
18	0.444	27.22	28.43	39	0.582	27.91	30
19	0.446	27.23	28.44	40	0.585	27.93	30
20	0.461	27.31	28.64	41	0.624	28.12	30
21	0.467	27.34	28.93	42	0.764	28.82	30

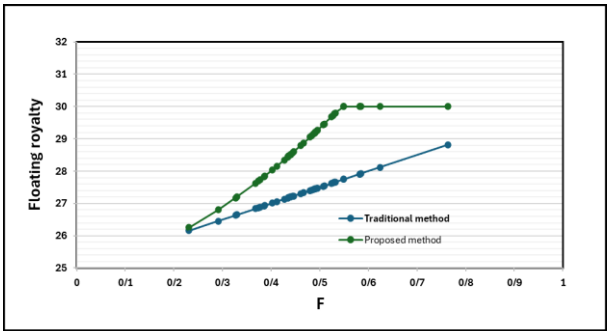

Fig. 2. Comparison of results between traditional and proposed methods

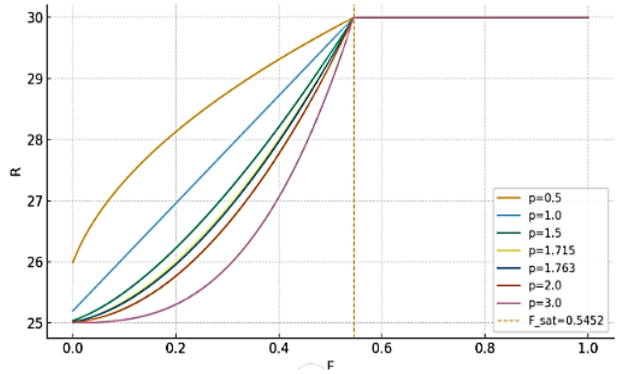

Fig. 3. Effect of parameter (p) on (R)

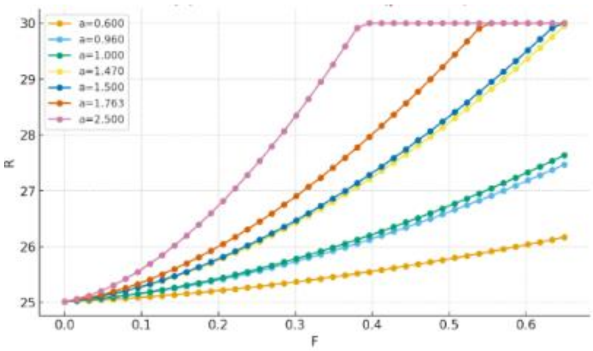

Fig. 4. Effect of parameter (a) on (R)

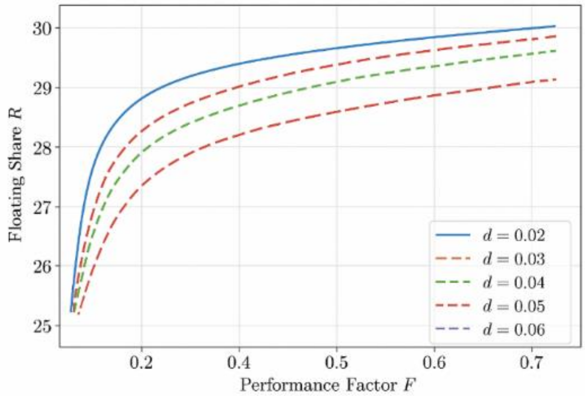

Fig. 5. Effect of parameter (d) on (R)

Considering that parameter (d) only causes a limited shift in the initial value of the function, its effect on the floating-share was examined over the range of 0.02 to 0.06, and the corresponding trend is presented in Fig. 5. Based on the calculations and analyses conducted, the optimal value of (d) is 0.0379.

Considering the gold price fluctuations during the first half of 2025 and using the calculated (F) values for each month, the seven aforementioned scenarios were solved, and the resulting outcomes are presented in Table 7.

Also, key indicators including mean floating-share price, standard deviation, risk, conservative profit of the mine owner, coefficient of variation (CV), profit percentage relative to the best scenario, and the composite decision-making index (CI)— have been used to evaluate and compare the results obtained from different scenarios. The aforementioned relationships are presented in Tables 8 and 9.

Table 5. Proposed equations for different scenarios

Scenario Number	Proposed equations
1	$R = R_{min} + (R_{max} - R_{min}) \times F$
2	$R = R_{min} + (R_{max} - R_{min}) \times \min((1.283F + 0.019)^{2.6}, 1)$
3	$R = R_{min} + (R_{max} - R_{min}) \times \min((1.1.3624F + 0.0415)^{1.48}, 1)$
4	$R = R_{min} + (R_{max} - R_{min}) \times \min((1.3154F + 0.0745)^{0.8624}, 1)$
5	$R = R_{min} + (R_{max} - R_{min}) \times \min((1.4718F + 0.0417)^{1.6}, 1)$
6	$R = R_{min} + (R_{max} - R_{min}) \times \min((1.603F + 0.0403)^{1.75}, 1)$
7	$R = R_{min} + (R_{max} - R_{min}) \times \min((1.762F + 0.0379)^{1.95}, 1)$

Table 6. Computational parameters in different scenarios

Parameters	Scenarios						
	1	2	3	4	5	6	7
ε	--	0.05	0.25	0.5	0.25	0.25	0.25
F_{tmin}	0.231	0.231	0.25725	0.2835	0.25725	0.25725	0.25725
F_{tmax}	0.764	0.764	0.7035	0.7035	0.651	0.5985	0.546
r	--	0.3023	0.3656	0.4029	0.3951	0.4298	0.423
a	--	1.283	1.3624	1.3154	1.4718	1.603	1.762
d	--	0.019	0.0415	0.0745	0.0417	0.0403	0.0379
p	--	2.6	1.48	0.8624	1.6	1.75	1.95

Table 7. Results of different scenarios for calculating the floating-share

Date	F	Scenarios						
		1	2	3	4	5	6	7
20/04/2025	0.527	27.635	26.942	28.327	28.833	28.621	29.038	29.678
21/05/2025	0.585	27.925	27.530	28.853	29.320	29.245	29.810	30
20/06/2025	0.291	26.455	25.439	26.473	27.546	26.494	26.522	26.562
21/07/2025	0.490	27.450	26.616	28.006	28.762	28.243	28.577	29.083
20/08/2025	0.507	27.535	26.761	28.152	28.863	28.414	28.786	29.351
20/09/2025	0.461	27.305	26.385	28.347	28.589	27.957	28.231	28.643

Table 8. Key computational metrics

Metric	Description	Formula	Eq.
Mean	Average owner profit across periods	$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$	(22)
Standard Deviation (Std. Dev)	Dispersion of data around the mean	$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$	(23)
Normalized Risk (0-1)	Risk scaled between 0 and 1	$R = \frac{\sigma}{\sigma_{max}}$	(24)
Conservative Owner Profit	Conservative profit estimate for the mine owner	$Profit_{cons} = \bar{X} - \sigma$	(25)
Coefficient of Variation (CV)	Relative risk with respect to mean profit	$CV = \frac{\sigma}{\bar{X}}$	(26)
Composite Index (CI)	Combination of profit and risk for scenario prioritization	$CI = 0.7 \times \frac{Profit_{conservative}}{Profit_{best}} + 0.3 \times (1 - R)$	(27)
Alignment with Reference Scenario (%)	Comparison of scenario profit with reference scenario	$Alignment\ (\%) = \left(1 - \frac{\lvert Profit - Profit_{reference}\rvert}{Profit_{reference}}\right) \times 100$	(28)

Table 9. List of parameters used in Table 8

Explanation	Parameter
X_i	Mine owner profit in period I (each period contributes one data point)
n	Number of periods considered in the analysis
$\bar{X}$	Mean (average) mine owner profit across all periods
σ	Standard deviation of owner profit (represent volatility)
σ_{max}	Maximum standard deviation among all scenarios (used for risk normalization)
R	Normalized risk of the scenario (0-1 scale)
$Profit_{cons}$	Conservative owner profit, calculated as mean minus standard deviation
$Profit_{best}$	Highest conservative profit among all scenarios
CV	Showing relative variability
CI	Composite index, combining profit and risk
alignment	Percentage alignment of scenario profit with the reference scenario
priority	Scenario ranking based on CI or combined profit and risk

The results presented in Table 10 indicate the following:

- Scenario 4 was identified as the optimal scenario, providing the highest conservative profit for the mine owner (28.109), while its risk level remains in the medium range (0.503).
- Compared to Scenario 4, the other scenarios exhibit profit differences ranging from 0.053 to 1.502 units, with their correspondence to the reference scenario between 94.7% and 99.8%; therefore, all scenarios are considered relatively close and stable in terms of the mine owner's profit.
- The conservative profit of the mine owner (mean minus standard deviation) suggests that scenarios with lower volatility have a higher capability to guide conservative economic decision-making.
- The composite index (CI), with a weighting of 0.7 for profit and 0.3 for (1–risk), allows for a logical ranking of the scenarios, indicating that high profit combined with controlled risk is the most important factor in selecting the optimal scenario.
- The scenario trends are generally oscillatory, with temporary decreases or recoveries; however, none of the scenarios experience a persistent or severe decline, indicating that all scenarios are practically feasible.

Table 10. Ranking of different scenarios

Priority	Scenario	Mean	Std. Dev	Risk (0-1)	Owner Profit	CV	Profit % of Best	CI	Alignment with Scenario 4(%)	Stability	Volatility
1	4	28.652	0.543	0.503	28.109	0.019	100	1	100	High	Medium
2	7	28.886	1.079	1	27.807	0.037	98.9	0.89	97.2	Medium	High
3	6	28.494	0.828	0.768	27.666	0.029	98.4	0.91	98.6	Medium	Medium
4	5	28.162	0.824	0.764	27.338	0.029	97.3	0.88	99.8	Medium	High
5	3	28.026	0.740	0.686	27.286	0.026	97.1	0.90	99.7	Medium	Medium
6	1	27.384	0.456	0.423	26.928	0.017	95.8	0.93	97.4	High	Low
7	2	26.612	0.632	0.586	25.980	0.024	92.4	0.87	94.7	Medium	Medium

- Overall, Scenario 4 is selected as the optimal reference, and the other scenarios, with minor differences in profit and risk, are considered acceptable options. This analysis provides a basis for optimizing the mine owner's economic decision-making with respect to both profit and risk.

V. Conclusion

This study demonstrated that dynamic modeling combined with the use of nonlinear adjustment coefficients is an effective tool for optimizing the floating-share in gold mines. The analysis results indicated that this approach provides greater flexibility in coping with severe price fluctuations compared to traditional linear methods, and capable of maintaining the floating-share consistently above the minimum and often near its maximum value. Furthermore, the sensitivity analysis revealed that the parameter (p) plays a key role in determining the curvature and growth rate of the floating-share function, while parameters (a) and (d) primarily regulate the initial slope and position of the function. Therefore, precise design of adjustment coefficients and continuous market monitoring are essential for achieving optimal performance. Ultimately, the proposed model not only enables optimal decision-making and risk management under price volatility but also offers a generalizable approach applicable to other mines and mineral resources with similar characteristics.

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