Journal of Geomine (2025), 3(4), 215-224
DOI: 10.22077/jgm.2026.10549.1064

Field-Based Assessment of Analytical and Statistical Grout Take Models: Case Studies

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Recived: 2025 Nov. 22, Revised: 2026 May 06, Accepted: 2026 May 11, Published: 2026 Jun. 04

ABSTRACT

Accurate estimation of cement take is crucial for the economical and efficient design of grout curtains in dam foundations. This research aimed to compare the accuracy of common analytical models and to develop new models based on real-world data from grouting operations at the Azad Kurdistan and Siah Bishe dams. Initially, seven well-known analytical models (Johnson, Stille, Hassler, Lombardi, Håkansson, Bitobi, and Rastegarnia) were evaluated. The results indicated that the Hassler model, with relative errors of 6.8% and 12.9% for the Azad and Siah Bishe dams, respectively, performed best among the analytical models. Subsequently, to enhance accuracy, regression analysis was employed using SPSS software. For the Azad dam, where a linear relationship was prevalent, a multiple linear regression model was developed, achieving an adjusted coefficient of determination (R^2) of 0.985 and a Root Mean Square Error (RMSE) of 0.008. For the Siah Bishe dam, which exhibited nonlinear behavior, a third-degree multiple nonlinear regression model was developed, showing a coefficient of determination of 0.917 and an RMSE of 0.24. Finally, a comparison of evaluation metrics confirmed the significant superiority of the developed statistical models over the best analytical model (Hassler). This study underscores the strong capability of data-driven statistical approaches for predicting cement take based on field data.

KEYWORDS

Dam sealing, grouting, cement take, analytical models, linear and nonlinear regression, statistical analysis

I. INTRODUCTION

The safety and stability of reservoir dams depend directly on controlling water seepage through their foundations and abutments. One of the most effective methods to achieve this is the construction of a cutoff wall through the injection of cement grout into rock mass fractures. The success of this operation, in addition to proper design, relies on the accurate estimation of cement take. An inaccurate estimation can lead to serious technical issues, such as an incomplete cutoff wall and continued seepage, as well as economic consequences, including unnecessary cost increases. To address this challenge, numerous analytical models including those developed by Johnson, Stille, Hassler, and Lombardi have been proposed. These models attempt to simulate the grout penetration process by simplifying the complexities of the grouting environment. However, the main weakness of these models lies in their inability to fully account for all relevant geotechnical and operational parameters across different sites, resulting in significant errors in actual projects (Nonveiller, 2016). This research aims to conduct a comparative evaluation of the accuracy of common analytical models and to develop new statistical models based on field data. The innovations of this study include:

- Systematic comparison of seven analytical models at two distinct geological sites
- Development of customized regression models for each project
- Provision of a practical data-driven approach for engineers

II. Parameters Affecting Cutoff Wall Design and Grouting Operations

The optimal design of cutoff walls and the successful execution of grouting operations require a comprehensive understanding of the influential parameters and careful consideration of their complex interactions. These parameters can be classified into three main categories (Hao et al., 2023):

A. Grouting Environment Parameters (Geotechnical)

This category encompasses the inherent conditions of the project site and represents the most critical factors determining rock mass behavior during grouting (U.S. Army Corps of Engineers, 2010).

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- Geomechanical Characteristics of the Rock Mass:
  - o Rock Quality Designation (RQD)
  - o Joint system (orientation, persistence, spacing, and aperture)
  - o Uniaxial compressive strength of intact rock
  - o Weathering and alteration state
- Hydrogeological Conditions:
  - o Groundwater level
  - o Hydraulic gradient
  - o Initial permeability (Lugeon Value)
  - o Groundwater flow pattern
- Site-Specific Conditions:
  - o Bedrock depth
  - o Dip of layers and joints
  - o Presence of crushed zones or faults

B. *Grout Mix Parameters*

The rheological properties of the grout play a decisive role in the penetration mechanism and the filling of voids (Soleymani and Akhtarpur, 2011):

- Water-to-Cement Ratio (W/C)
- Viscosity and specific gravity
- Yield Stress
- Initial and final setting times
- Bleeding and stability of the suspension

C. *Operational Grouting Parameters*

These parameters are directly controlled by the operator and can be adjusted during the operation (Eftekhari and Aalianvari, 2019):

- Grouting Pressure:
  - o Maximum allowable pressure
  - o Pressure increase rate
  - o Pressure maintenance duration
- Geometric Specifications:
  - o Grouting stage length
  - o Borehole diameter
  - o Spacing between boreholes
- Execution Method:
  - o Up-stage or Down-stage grouting
  - o Use of packers
  - o Number of grouting cycles

D. *Quality Control Parameters*

- Grouting flow rate (L/min)
- Grout take per stage
- Pressure-volume behavior during grouting
- Results of post-grouting quality control tests

Fig. 1: Classification of parameters affecting the design and execution of cutoff walls

A correct understanding of these parameters and their interrelationships forms the fundamental basis for selecting the appropriate grouting method, optimizing the cutoff wall design, and accurately predicting cement take. The following section reviews common analytical models that attempt to quantify these relationships (Ewert, 2012).

III. INVESTIGATION OF ANALYTICAL AND STATISTICAL METHODS FOR ESTIMATING GROUT TAKE

A. *Analytical Methods*

1) *3.1.1. Theoretical Framework and Fundamental Assumptions*

Analytical models for grout take estimation are fundamentally based on simplified representations of grout flow through fractured rock masses. These models typically employ rheological principles adapted to geological environments, with most sharing several key assumptions. The primary theoretical foundation assumes Bingham plastic behavior for cement-based grouts, where the material exhibits a distinct yield stress before flowing as a viscous fluid. Additional common premises include treating fractures as parallel plates with constant aperture, assuming laminar flow under steady-state conditions, and considering the rock mass to be macroscopically isotropic. While these simplifications enable mathematical tractability, they inevitably limit the models' ability to capture the full complexity of field conditions, particularly in heterogeneous geological settings (Sarkarabad et al., 2022).

2) *Comprehensive Review of Analytical Models*

This research conducted a systematic evaluation of seven prominent analytical models, each representing a distinct theoretical approach to grout take estimation:

a) *Johnson Model*

This pioneering model conceptualizes fractures as disk-shaped features with a uniform aperture distribution. The fundamental equation (Nouri and Salmasi, 2017):

$$I = \frac{\Delta P . b}{2\,\tau_0} \tag{1}$$

Where I represents the penetration radius, ΔP denotes the pressure differential, b indicates the fracture aperture, and τ signifies the grout yield stress, this equation forms the basis for many subsequent developments. The model's strength lies in its mathematical simplicity but is limited by an oversimplification of fracture geometry. The volume of grout required for injection into the disk in question is equal to:

$$V = I^2 . b . \pi \tag{2}$$

Where:

- V = Grout volume
- I = Radius of grout penetration
- b = Fracture aperture
- π = Mathematical constant p_i (approximately 3.1416)

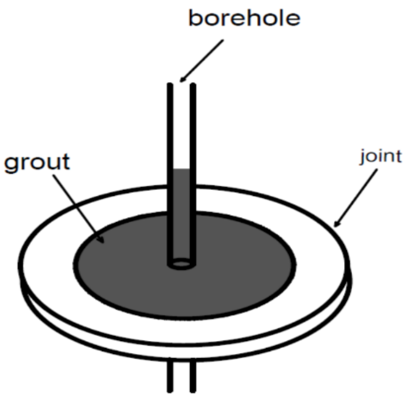

Fig.1. Schematic depiction of geometrical mode of the join

This equation assumes:

1. A disk-shaped fracture with constant aperture
2. Complete filling of the fracture void
3. Uniform grout penetration in all directions
4. No grout loss or filtration effects

The volume represents the theoretical minimum requirement and should be adjusted for field conditions by considering factors such as (Eftekhari et al., 2018):

- Grout bleed and shrinkage
- Fracture connectivity and complexity
- Groundwater flow effects
- Practical execution tolerances

And for N joint, the equation is as follow:

$$V = (\frac{\Delta P}{2.\tau_0})^2 . N . b^3 . \pi \quad (3)$$

b) Stille Model

In this model, it is assumed that grout is injected from the borehole into a network of fractures and flow paths within the rock, spreading at an angle of α. The ideal grout spread angle within a fracture is $\alpha = 2\pi$.

Wallner (1976) and Lombardi (1985) estimated the radius of grout influence using the following relationship (Terzaghi, 1943):

$$I = \frac{\rho_w\, g\, (h_w - h)}{2.\tau_0} b \quad (4)$$

Where:

- I: Length of the injected section
- $\rho_w g(h_w - h)$: Pressure difference between groundwater pressure and grouting pressure
- τ_0: Yield stress of the grout
- b: Fracture aperture

The volume of injected grout, assuming the presence of a single fracture, can be estimated using Eq. (5) (Bennet, 1946):

$$V = I^2 . b . \frac{\alpha}{2} \quad (5)$$

Assuming the existence of N injectable fractures, the total grout volume can be calculated with Eq. (6) (Metheron, 1963):

$$V = I^2 . b . \frac{\alpha}{2} . N \quad (6)$$

c) Hassler Model

In this model, a disk-shaped fracture perpendicular to a borehole passing through its center is assumed. The parameter representing the grouting borehole perimeter (WW) also plays a significant role in estimating the grout take. The grout penetration radius in this model is obtained from Eq. (7) (Tayfur et al., 2005):

$$I = \frac{\Delta P . b}{2\tau_0} \quad (7)$$

In Hassler's model, the injected grout volume is expressed as a function of the geometric parameters of the rock, as follows (Richards and Reddy, 2007):

$$V = \frac{\Delta P}{2\tau_0} b2 \left(W + \frac{\Delta P\ b}{2\tau_0} \times \frac{\alpha}{2} \right) \quad (8)$$

Here, W is the borehole perimeter, and α is the grout spread angle.

d) Lombardi Model

In this model, the grout penetration radius is similar to that in previous models. Lombardi introduced the injected grout volume per meter of borehole using the following equation, where L is the length of the grouting section (Fig. 2) (Nouri and Salmasi, 2017):

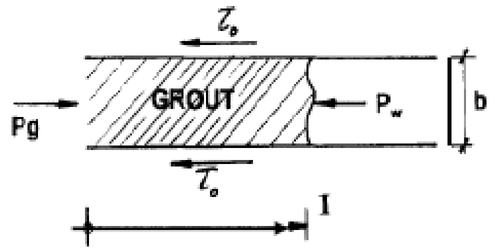

Fig. 2. Grout penetration into a fracture

$$I = \frac{P.b}{2\tau_0} \quad (9)$$

$$V = I^2 . b . \frac{L}{2} \quad (10)$$

e) Håkansson and Hassler Model

This model assumes that the apertures of fractures in the rock are not constant. This condition can be simulated by assuming the fracture aperture remains constant over fixed intervals and increases step-by-step. Fig. 3 illustrates this scenario.

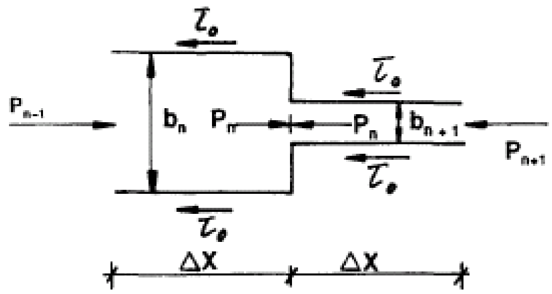

Fig. 3. Fracture with variable aperture

The constant fracture aperture at each stage is derived from the following equation:

$$P_{n-1}.\, b_n = 2\,\tau_0.\, \Delta x + P_n\, .B_n \quad (11)$$

$$P_0 = P_n - 2\,\tau_0\, \Sigma\, (\Delta x\, /\, b_n) \quad (12)$$

$$I = [(Pg - Pw)\, /\, 2\,\tau_0]\, .\, [N/\Sigma\, (1/\, bn)] \quad (13)$$

According to several authors, the harmonic mean is calculated as follows:

$$b_{hm} = \exp\, (\mu_y - \sigma^2{}_y/2) \quad (14)$$

Here, b_{hm} is the harmonic mean of the apertures. The following equations for the penetration radius and the injected grout volume are presented in this model:

$$I_{max} = (\Delta P/2\,\tau_0)\left(\sum b^3\right)^{1/3} \quad (15)$$

$$V_{grout} = (\Delta P/2\,\tau_0)^2 .\pi\, .\sum\, b^3 \quad (16)$$

IV. Statistical Analysis of Cement Grout Take

Given that not all parameters obtained from grouting boreholes influence analytical models used to estimate grout take, and that some models include parameters whose values cannot be determined from geotechnical data, drilling cores, or tunnel grouting records, it is crucial to analyze the actual relationships among all existing and effective parameters in grouting operations and to assess their impact and significance. Consequently, this study aims to employ statistical analysis alongside the evaluation of analytical models.

A. Regression

Regression analysis is a statistical technique used to investigate and model relationships between variables. It is considered one of the most widely used methods in statistical analysis. The relationships between parameters can be either linear or nonlinear. In the following sections, various types of relationships between parameters are examined through linear and nonlinear regression models.

B. Linear Regression

In general, if a correlation pattern can be expressed as a linear equation, it is called a linear regression equation. Its purpose is to predict the behavior of the dependent variable based on the values of the independent variables. In this research, the independent parameters are the depth of grouting sections, rock mass water absorption measured by the Lugeon test, water-to-cement ratio, fracture aperture, and grouting pressure. The dependent parameter is the volume of cement grout taken.

In general, if x is the independent variable and y is the dependent variable, the linear regression equation $y=ax+by=ax+b$ is established between the two variables. The value of R, known as the linear correlation coefficient, expresses the strength and direction of this linear relationship between the two variables. This parameter is calculated using Eq. (17):

$$R = n\Sigma xy - (\Sigma x)(\Sigma y)/(\sqrt{(n\Sigma x^2}) - (\Sigma x)^2)(\sqrt{n(\Sigma y^2)} - (\Sigma y)^2) \quad (17)$$

In this equation, n is the number of data points. When there are multiple independent variables, the multiple linear regression equation is established to describe the relationship between the independent variables and the dependent variable as follows:

$$y = ax_1 + bx_2 + cx_3 + \ldots + \quad (18)$$

C. Nonlinear Regression

In many cases, researchers relate the dependent variable to independent variables using a mathematical expansion, which often involves nonlinear parameters. In such instances, the linear regression method must be extended, introducing considerable complexity.

The nonlinear regression relationship can be expressed as follows:

$$Y = f(xn,\, \theta) + Z_n \quad (19)$$

Here, f is the expectation function, and x_n is a vector containing the regression or independent variables for the nth case. The key difference is that the expected response is a nonlinear function of the parameters; to emphasize the difference between linear and nonlinear patterns, θ is used as the nonlinear parameters. As before, PP is also considered for a number of parameters. When analyzing a specific dataset, the vector xn is considered fixed and centered on the dependence of the expected responses on θ. An N-vector ($\eta(\theta)$) with n elements is constructed as shown in the following Eq.:

$$\eta(\theta) = f(x_n,\theta) + Z,\ n = 1\ldots N \quad (20)$$

And the nonlinear pattern is considered as the following equation:

$$Y = \eta(\theta) + Z \quad (21)$$

Here, it is assumed that ZZ follows a normal, spherical distribution, similar to the linear pattern, that is:

$$Var(Z) = E[ZZT] = \sigma_2 I,\ E[Z] = 0 \quad (22)$$

Foundational analytical frameworks for simulating grout flow in fractured media, as presented in key studies, as summarized in Table 1.

V. Case Study

To validate the introduced models and estimate the volume of cement grout take using these methods, drilling and grouting data from the Azad and Siah Bishe dam sites were used. These sites are described below.

Table 1. Comparative summary of analytical models for grout take estimation

Researcher(s) (Year)	Model Name / Key Concept	Key Assumptions	Main Contribution / Application
Johnson	Radial Flow (Disk Model)	Single, disk-shaped fracture with constant aperture (*b*). Laminar flow of Bingham fluid.	Foundational model. Introduces the basic relationship between pressure, aperture, yield stress, and penetration radius.
Stille & Wallner	Spread Angle Concept	Grout spreads with a specific angle (α) from the borehole into a network of fractures.	Introduced the grout spread angle (α), providing a more realistic geometric representation of grout flow in multiple fractures.
Lombardi	Linear Pressure-Volume	Emphasizes the role of grouting stage length (L). Simple linear relationship.	Practical model for preliminary design estimates, focusing on the volume per meter of the borehole.
Hassler	Borehole Perimeter	Considers the geometry of the injection source (borehole perimeter, W).	Incorporated the borehole perimeter (W) to account for the initial geometry of grout spread around the injection point.
Håkansson & Hassler	Variable Aperture & Harmonic Mean	Fracture aperture is not constant but variable. Uses harmonic mean for aperture.	Addressed a key limitation by modeling variable fracture apertures, leading to a more realistic representation of natural rock masses.
Jacobi, Raffi & Stille	Grouting Intensity Number (GIN)	Relates injection pressure (P) and volume (V) through an energy concept (GIN).	Introduced the GIN method, which provides a practical framework for controlling grouting operations by limiting the combined energy of pressure and volume.
Rastegarnia	Effective Penetration Length	Based on Gustafson & Stille; focuses on effective penetration length and average	

A. Azad Dam Site

The primary objective of constructing the Azad Dam is to supply water for the development of agricultural lands in the Ravansar Plain by building a reservoir on the Kumasi River, a main tributary of the Azad River (the primary headwater of the Sirwan River). It is situated in the west and northwest of Sanandaj County in Kurdistan Province. Azad Dam is a rockfill structure with a clay core. It has a height of 115 meters from the foundation, and the diversion system includes a tunnel approximately 504 meters long. The dam's spillway is a free overflow type, located on the left abutment (Fig. 4).

B. Siah Bishe Dam Site

The Siah Bishe Dam and Power Plant are located 7 km north of Tehran and 10 km north of the Kandovan Tunnel, along the Chalus River. It includes a pumped-storage power plant with a total capacity of 1,000 MW, consisting of four units of 250 MW each. This project comprises two dams and one pumped-storage power plant. The pumped-storage plant generates electricity during peak demand hours and, during periods of low demand, pumps water to the upper reservoir for storage. This process consumes electricity but helps regulate the grid load (Fig. 4).

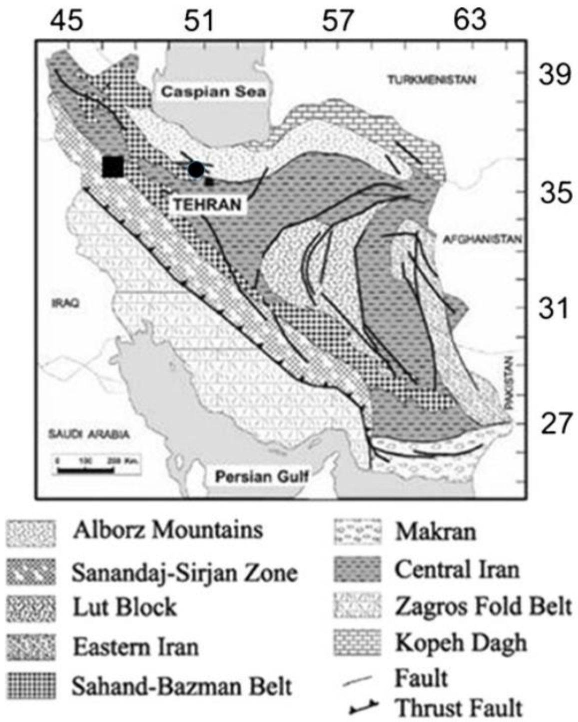

Fig. 4. Azad powerplant(black square) and Siahbishe powerplant(black circle) on Iran geological map

C. Calculation of Grout Take Based on Analytical Relations

In calculating grout take based on analytical relations, data from grouting sites were used. First, the radius of penetration was obtained using the introduced analytical relation. Then, using the injection radius and other necessary parameters, the grout take volume was calculated. Subsequently, the relative error in estimating the cement grout take (comparing the actual grout take volume with the calculated grout take volume) was computed using the following formula. The results are presented in Table 2 and Fig.s 6 and 7.

$$e=[(V_R-V_c)/ V_R].100 \quad (23)$$

In this relation, e is the percentage of relative error, V_R is the actual grout volume, and V_C is the calculated grout volume.

Interpretation and Key Findings:

1. Best Performing Model: The Hassler model is clearly the most accurate for both dams, with a remarkably low relative error of 6.809% for Azad Dam and 12.929% for Siah Bishe Dam. Its performance on Azad Dam is exceptionally strong.

Table 2. Relative Error Percentage of Analytical Models in Estimating Grout Take (%)

Model	REP (%) Azad Dam	REP (%) Siah Bishe Dam
Rastegarnia	98.1982	98.639
Jacobi	98.400	98.506
Håkansson	62.765	98.500
Lombardi	99.005	98.811
Hassler	6.809	12.929
Stille	99.622	99.748
Johnson	97.720	98.506

2. Worst Performing Models: The Stille and Lombardi models consistently showed the highest errors, above 99% and 98.8%, respectively for both dams. This indicates a very poor fit to the site data.

3. Inconsistent Performance: The Håkansson model performed moderately well on the Azad Dam (62.77% error) but its accuracy decreased significantly for the Siah Bishe Dam (98.5% error), similar to other lower-performing models. This indicates that its performance is highly site-dependent.

4. Consistently High Error Models: The Johnson, Jacobi, and Rastegarnia models all yielded consistently high errors (around 97.7% to 98.6%) for both dams, indicating a systematic lack of accuracy in this context.

Given the geological conditions at the Azad and Siah Bishe dams, the Hassler model is the most reliable analytical method for predicting grout take, significantly outperforming all other models evaluated in this study.

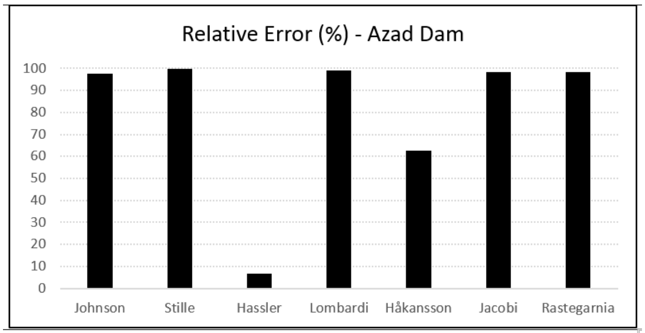

Fig. 5. Relative Error Percentage of Analytical Models for Azad Dam

Fig. 5 illustrates the relative error percentages of seven analytical models in estimating the grout take for the Azad Dam case study. The Hassler model demonstrates a significantly lower error (6.81%) compared to all other models, which exhibit errors above 60%. The Håkansson model shows a moderate error (62.77%), while the remaining five models (Johnson, Stille, Lombardi, Jacobi, and Rastegarnia) display very high errors, closely clustered between 97% and 100%.

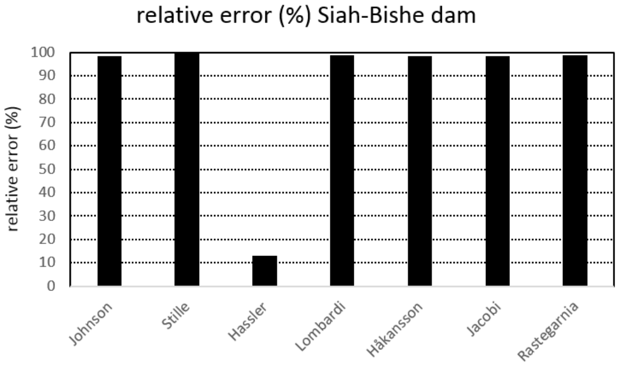

Fig. 6. Relative Error Percentage of Analytical Models for Siah Bishe Dam

Fig. 6 presents the relative error percentages of the same analytical models for the Siah Bishe Dam case study. The Hassler model remains the most accurate, exhibiting a substantially lower error rate (12.93%) compared to the alternatives. In contrast to the Azad Dam results, the other six models perform very poorly and with remarkably similar effectiveness, showing errors ranging from 98.5% to 99.75%.

Based on the computational error results shown in Figures 5 and 6, the Hassler model is identified as the most accurate estimator for the cement grout volume in both the Azad and Siah Bishe dams, with error rates of 6.8% and 12.9%, respectively.

1) ***Evaluation Metric for Azad Dam***

$$RMSE = \sqrt{[\Sigma(Vpred - Vmeas)^2/N]} = 0.056$$

2) ***Evaluation Metric for Siah Bishe Dam***

$$RMSE = \sqrt{[\Sigma(Vpred - Vmeas)^2/N]} = 0.028$$

To achieve the most accurate refined model for estimating cement grout take, Eq. (24) was proposed for Azad Dam, and Eq. (25) for Siah Bishe Dam.

$$V = (I \cdot b/0.93191) \cdot (W + I \cdot \alpha/2) \quad e = 0.0006\% \tag{24}$$
$$V = (I \cdot b) \cdot [(W/0.1017) + (I \cdot \alpha/2)] \quad e = 0.003\% \tag{25}$$

These refined models demonstrate significantly improved accuracy, with errors reduced to as low as 0.0006% and 0.003%, respectively.

3) Calculation of Grout Take Based on Statistical Analysis

In this analysis, the grouting data obtained from the grouting sites were first entered into the software as input. Then, statistical analysis was conducted to identify the dependent parameter (grout take volume) and the independent parameters (other variables). Linear regression was applied to the data from the Azad Kurdistan Dam due to the linear nature of the relationships, while nonlinear regression was used for the Siah Bishe Dam because of the nonlinear relationships.

4) Evaluation Metrics

The coefficient of determination (R^2) and the root mean square error (RMSE) were used to evaluate the models, calculated using Eq.s (26) and (27):

$$R^2=1-\sum(X_i-Y_i)^2/\sum(X_i-\bar{X})^2 \quad (26)$$
$$RMSE=\sum(X_i-Y_i)^2/N \quad (27)$$

In these equations, X_i is the observed value at the i-th time step, Y_i is the calculated value at the same time, N is the total number of time steps, and $\bar{X}$ is the mean of the observed values. A low RMSE value and a high R^2 value can indicate the accuracy of a model compared to competing models.

5) Linear Regression Calculations

The grouting data from the Azad Dam site were divided into four separate boreholes, each analyzed independently. Linear regression was applied to these boreholes using both forward and backward stepwise methods. The correlation between the independent and dependent variables was determined using the regression coefficient (R) and its square (R^2). The significance value (p-value) of the regression model was evaluated to determine whether the model is a reliable predictor of grout take volume. Finally, a model in the form of Eq. (28) is proposed to estimate the grout take volume.

$$V=a.D + b.L_u + c.RQD + d.P + f.(W/C) + g.b + h.\ \tau_0 + I \quad (28)$$

Where:

- D: Depth
- L_u: Lugeon value
- RQD: Rock Quality Designation
- P: Grouting pressure
- W/C: Water-cement ratio
- b: Fracture aperture
- τ_0: Grout yield stress
- i: Intercept constant

For the Azad Kurdistan Dam, the adjusted coefficient of determination (R^2) in linear regression showed little difference compared to that in nonlinear regression. Therefore, linear regression was used for this dam.

Borehole 1 of Azad Kurdistan Dam:

In Borehole 1, the backward method was used. According to the results obtained, a linear regression model was employed for this borehole, with an adjusted coefficient of determination (R^2) of 0.986, a significance level (Sig.) of 0.003, and the influence coefficients of the independent variables on the dependent variable. The results indicate that three variables—grout cohesion (τ_0), rock quality designation (RQD), and fracture aperture (b)—with coefficients B of 0.517, 0.500, and 0.232, respectively, have a significant influence on the dependent variable, namely, the grout take volume. Other parameters were excluded from the linear regression model due to their lack of statistical significance (Fig. 7).

- Relative error percentage for Borehole 1 of Azad Dam:

Relative error percentage for Borehole 1 of Azad Dam
$e=3/44$
Evaluation metric for Borehole 1 of Azad Dam

$$RMSE=\sqrt{(\Sigma(V_{\text{pred}}\text{-}V_{\text{meas}})^2/N)}=0.003$$

Finally, the model for Borehole 1 of Azad Dam was presented as follows (Eq. (28)):

$$V=\text{-}0.606+0.5RQD+0/232b+0.517\tau_0,\ R^2=0.986 \quad (28)$$

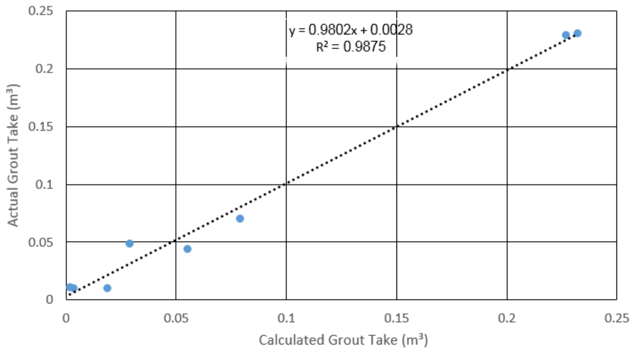

Fig. 7. Correlation between Measured and Predicted Grout Take for Borehole 1 at Azad Dam

Borehole 2 - Azad Kurdistan Dam:

In Borehole 2, the forward method was employed. The results from this borehole were analyzed using a linear regression model, which yielded an adjusted coefficient of determination (R^2) of 0.824 and a significance level (Sig.) of 0.002. The analysis of the influence coefficients of the independent variables on the dependent variable revealed that grout cohesion (τ_0), with a B coefficient of 2.922, had a significant impact on the grout take volume. Other variables were excluded due to lack of significance.

- Evaluation metric for Borehole 2 of Azad Dam:

$$RMSE=\sqrt{(\Sigma(V_{\text{pred}}\text{-}V_{\text{meas}})^2/N)}=0.039$$

The final model for Borehole 2 of Azad Dam was presented as follows (Eq. (29)):

$$V=-1.914+2.922\cdot\tau_0 \quad (R2=0.824) \quad (29)$$

Borehole 3 - Azad Kurdistan Dam:

In Borehole 3, the forward method was used. The results applied a linear regression model with an adjusted R^2 of 0.842 and a significance level (Sig.) of 0.028. The influence coefficients indicated that grout cohesion (τ_0), with a B coefficient of 1.751, had a significant effect on the grout take volume. Other variables were excluded due to their lack of significance.

- Evaluation metric for Borehole 3 of Azad Dam:

$RMSE=\sqrt{\Sigma(V_{pred}-V_{meas})^2/N}=0.023$

The final model for Borehole 3 of Azad Dam was presented as follows (Eq. (30)):

$$V=-1.119+1.751\cdot\tau_0\ (R^2=0.842) \quad (30)$$

Borehole 4 - Azad Kurdistan Dam:

In Borehole 4, the backward method was applied. The results from this borehole were analyzed using a linear regression model with an adjusted R^2 of 0.985 and a significance level (Sig.) of 0.031. The influence coefficients demonstrated that four variables—grout cohesion (τ_0), rock quality designation (RQD), grouting pressure (P), and fracture aperture (b)—with coefficients (B) of 0.12, 0.53, 0.38, and 0.24, respectively, had a significant impact on the grout take volume (Fig. 8).

- Evaluation metric for Borehole 4 of Azad Dam:

$RMSE=\sqrt{\Sigma(V_{pred}-V_{meas})^2/N}=0.008$

The final model for Borehole 4 of Azad Dam was presented as follows (Eq. (31)):

$$V=-1.483+0.478\cdot RQD-1.098\cdot P-3.549\cdot b+3.149\cdot\tau_0\ (R^2=0.985) \quad (31)$$

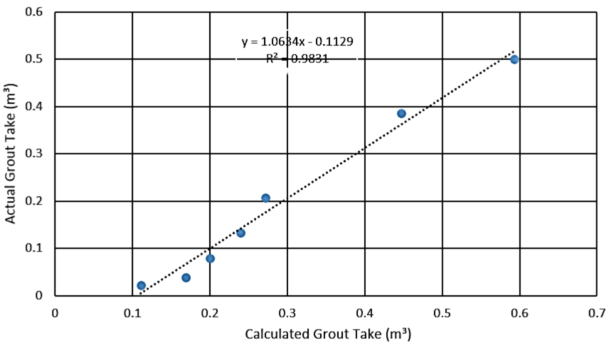

Fig. 8. Correlation between measured and predicted grout take for Borehole 4 at Azad Dam

D. *Nonlinear Regression Analysis*

The relationship between variables can typically follow linear, logarithmic, parabolic, exponential, inverse, power, growth, or other patterns. Scatter plots revealed that the relationship between grouting parameters at the Siah Bishe Dam is nonlinear. Therefore, nonlinear regression was applied to the grouting data from Siah Bishe Dam using SPSS24. The beta parameter indicates the share or weight of each independent variable in explaining the variance of the dependent variable. The general rule is that variables with a beta value not significant at the 0.05 error level are excluded from the model. Typically, B is used instead of beta, and a higher B value indicates a stronger relationship between variables. Siah Bishe Dam has three grouting boreholes, each analyzed separately. The adjusted coefficient of determination (R^2) for nonlinear regression at Siah Bishe Dam is better than for linear regression; hence, nonlinear regression was used.

Borehole 1 - Siah Bishe Dam

In Borehole 1, the independent variables are depth, rock quality coefficient, Lugeon value, pressure, and fracture aperture, while the dependent variable is grout take volume.

- *a*) Nonlinear Bivariate Regression: Depth and Grout Take

The independent variable is depth, and the dependent variable is grout take volume. The results showed an inverse function pattern with an adjusted R^2 = 0.863 and a significance level Sig. = 0.000. The regression equation is:

$V=0.005+0.033\cdot(1/Depth)$, $R^2=0.863$

- *b*) Nonlinear Bivariate Regression: Rock Quality Coefficient and Grout Take

The independent variable is the rock quality coefficient, and the dependent variable is the grout take volume. A quadratic function pattern was observed, with an adjusted R^2 = 0.729 and Sig. = 0.005. The regression equation is:

$V=4.189-10.175\cdot RQD+6.1\cdot RQD^2$, $R^2=0.729$

- *c*) Nonlinear Bivariate Regression: Lugeon Value and Grout Take

The independent variable is the Lugeon value, and the dependent variable is the grout take volume. A linear relationship was observed, with an adjusted R^2 = 0.863 and Sig. = 0.000. The regression equation is:

$V=0.929\cdot L_u V=0.929\cdot L_u$, $R^2=0.863$

- *d*) Nonlinear Bivariate Regression: Pressure and Grout Take

The independent variable is pressure, and the dependent variable is grout take volume. A cubic function pattern was observed, with an adjusted R^2 = 0.902 and Sig. = 0.001. The regression equation is:

$V=3.375-17.456\cdot P+28.75\cdot P^2-14.995\cdot P^3$, $R^2=0.902$

- e) Nonlinear Bivariate Regression: Fracture Aperture and Grout Take

The independent variable is fracture aperture, and the dependent variable is grout take volume. A linear relationship was observed, with an adjusted R^2 = 0.863 and Sig. = 0.000. The regression equation is:

$V=0.929\cdot L_u$, R2=0.863

Borehole 2 - Siah Bishe Dam

In Borehole 2, the independent variables are depth, Lugeon value, grouting pressure, and fracture aperture, while the dependent variable is grout take volume. The rock quality coefficient was excluded due to its high error.

- *a*) Nonlinear Bivariate Regression: Depth and Grout Take

Cubic function pattern with adjusted R^2 = 0.803 and Sig. = 0.007:

$V=0.176+0.038\cdot D-0.621\cdot D^2+0.581\cdot D^3$, $R^2=0.803$

- *b*) Nonlinear Bivariate Regression: Lugeon Value and Grout Take

Cubic function pattern with adjusted R^2 = 0.840 and Sig. = 0.004:

$V=0.049+0.069\cdot L_u+0.215\cdot L_u^2-0.153\cdot L_u^3$, $R^2=0.840$

- *c*) Nonlinear Bivariate Regression: Pressure and Grout Take

Cubic function pattern with adjusted R^2 = 0.852 and Sig. = 0.003:

$V=0.162+0.414\cdot P-0.926\cdot P^2-0.695\cdot P^3$, $R^2=0.852$

- *d*) Nonlinear Bivariate Regression: Fracture Aperture and Grout Take

Cubic function pattern with adjusted R^2 = 0.840 and Sig. = 0.004:

$V=0.049+0.069\cdot b+0.215\cdot b^2-0.153\cdot b^3$, $R^2=0.863$

Borehole 3 - Siah Bishe Dam

In Borehole 3, the independent variables are depth, Lugeon value, grouting pressure, and fracture aperture, while the dependent variable is grout take volume. The rock quality coefficient was excluded due to its high error.

- *a*) Nonlinear Bivariate Regression: Depth and Grout Take

Cubic function pattern with adjusted R^2 = 0.872 and Sig. = 0.004:

$V=0.028+1.151\cdot D-3.003\cdot D^2+1.968\cdot D^3$, $R^2=0.872$

- *b*) Nonlinear Bivariate Regression: Lugeon Value and Grout Take

Quadratic function pattern with adjusted R^2 = 0.823 and Sig. = 0.002:

$V=0.041+0.201\cdot L_u-0.61\cdot L_u^2$, $R^2=0.823$

- *c*) Nonlinear Bivariate Regression: Pressure and Grout Take

Cubic function pattern with adjusted R^2 = 0.856 and Sig. = 0.006:

$V=0.011+1.151\cdot P-2.676\cdot P^2+1.59\cdot$, $R^2=0.856$

- *d*) Nonlinear Bivariate Regression: Fracture Aperture and Grout Take

Quadratic function pattern with adjusted R^2 = 0.823 and Sig. = 0.002:

$V=0.041+0.201\cdot b-0.61\cdot b^2$, $R^2=0.823$

VI. Nonlinear Multivariate Regression

To derive a nonlinear multivariate relationship, the following steps were sequentially performed. After identifying nonlinear bivariate relationships, linear normalized data from grouting sites were incorporated, resulting in constant grout take volume parameters and updated values for the other involved parameters.

Borehole 1 - Siah Bishe Dam

Five independent variables were used in the analysis. The forward method was applied, yielding an adjusted R^2 = 0.937 and Sig. = 0.000. The influential variables identified were grouting pressure and fracture aperture, with beta coefficients of 0.57 and 0.445, respectively. Other parameters were excluded due to their lack of significance (see Fig. 9).

Evaluation metric: $RMSE=\sum(V_{pred}-V_{meas})2$, $N=0.02$

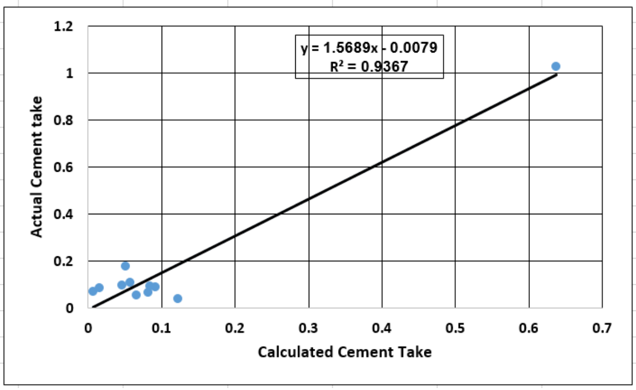

Fig. 9. Comparison between measured and calculated grout take volumes in Borehole 1 of Siah Bishe Dam using multivariate linear regression method

Final model: $V=0.576\cdot P+0.445\cdot b$, $R^2=0.961$

Borehole 2 and 3 - Siah Bishe Dam

Borehole 2 was excluded because its significance level exceeded 5% in the multivariate linear regression model. For Borehole 3, depth and fracture aperture showed a strong relationship, with an adjusted R^2 = 0.944 and Sig. = 0.001. However, the pressure parameter was excluded due to high error, and the fracture aperture parameter yielded illogical negative values, so Borehole 3 was also disregarded.

VII. Final Nonlinear Multivariate Regression Relationship

Using the nonlinear relationships between fracture aperture and grouting pressure from Borehole 1, the following final equation was derived:

$V=1.94 \cdot Pb-10.05 \cdot P+16.56 \cdot P^2-8.63 \cdot P^3+0.413 \cdot b$, $R^2=0.917$

Evaluation metric:

$RMSE=\sqrt{(\Sigma(V_{pred}-V_{meas})^2/N)}=0.34$

This final relationship, obtained through nonlinear multivariate regression, is the most suitable for this discussion. The comparison between actual and calculated grout take volumes shows close agreement, with an adjusted R^2 = 0.917 and Sig. = 0.000, as shown in Fig. 10.

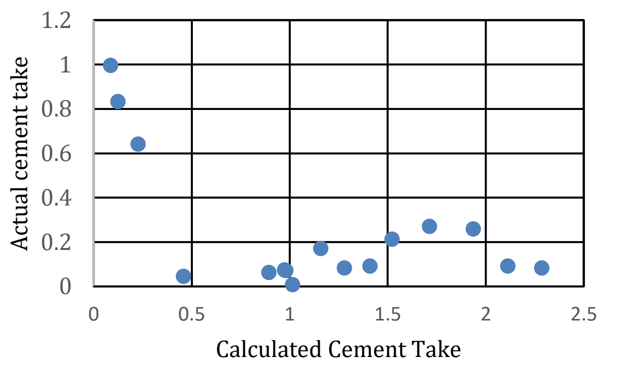

Fig. 10. Comparison between measured and calculated grout take volumes in Borehole 1 of Siah Bishe Dam using multivariate nonlinear regression method

VIII. Conclusions and Recommendations

- Analytical Models

The Hassler model is the most accurate analytical model for estimating grout take, with errors of 6.8% and 12.9% for the Azad and Siah Bishe Dams, respectively, and evaluation metrics of 0.05 and 0.02.

- Linear Regression

For Azad Dam, Boreholes 1 and 4, analyzed using backward linear regression, the most influential parameters are grout cohesion, rock quality coefficient, grouting pressure, and fracture aperture. The relative errors for these models are 3.44% and 4.08%, respectively, compared to 6.8% for the analytical model. The RMSE values are 0.003 and 0.008, significantly lower than the analytical model's 0.056. The adjusted R^2 values are 0.986 and 0.985, with significance values of 0.003 and 0.014, respectively. These results indicate the statistical models' superiority over the Hassler analytical model.

For Boreholes 2 and 3, analyzed using forward linear regression, grout cohesion is the most influential parameter. The adjusted R^2 values are 0.924 and 0.942, with Sig. values of 0.004 and 0.047, respectively. The RMSE values are 0.039 and 0.023, which are close to the Hassler model's RMSE of 0.028. These models are considered very good despite using only one parameter.

- Nonlinear Regression

For Siah Bishe Dam, nonlinear multivariate regression was applied to Borehole 1, resulting in a cubic nonlinear relationship. The most influential parameters identified were grouting pressure and fracture aperture. The evaluation metrics RMSE = 0.34, adjusted R^2 = 0.971, and Sig. = 0.000, indicate a well-fitted model.

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