Journal of Geomine (2025), 3(4), 201-214
DOI: 10.22077/jgm.2026.10236.1061

Mineral Exploration Potential Mapping through Fuzzy AHP Integration: A Case Study from the Kamoshgaran Region, Northwestern Iran

Ali Akbar Daya[1], Hadi Bejari[2]

Recived: 2025 Oct. 10, Revised: 2026 Feb. 21, Accepted: 2026 Apr. 08, Published: 2026 Jun. 30

ABSTRACT

The crucial initial step in any successful mineral exploration endeavor involves the careful selection of potential target areas, a process that requires consideration of a wide array of diverse and often interconnected criteria. To effectively navigate this complex decision-making landscape, researchers and exploration geologists frequently employ sophisticated spatial analysis techniques. Among these, the Analytic Hierarchy Process (AHP) modeling stands out as a particularly effective method for conducting studies that evaluate and map mineral potential. This study applies the Fuzzy Analytic Hierarchy Process (Fuzzy AHP) to produce a detailed potential map specifically for elemental mineralization within the Kamoshgaran region, northwestern Iran. The analysis meticulously considered several key criteria indicative of mineral deposit formation, such as underlying geology, geochemical signatures, hydrothermal alteration zones, and the presence of fault systems. Each criterion was rigorously evaluated using the Fuzzy AHP framework, allowing their relative importance to be determined through pairwise comparisons. A significant strength of this method lies in its ability to accommodate the integration of both quantitative data, such as geochemical assay values, and qualitative information, like geological mapping interpretations, within a unified analytical structure. Furthermore, the AHP framework inherently supports group decision-making, enabling the incorporation of expert knowledge and diverse perspectives from multiple stakeholders involved in the exploration process. Results showed that the central and eastern parts of the region have the highest ore mineralization potential; accordingly, detailed exploration activities should focus on these areas. According to the fuzzy method and the integration of these five important factors, their aggregate indicates that the central region is conditionally identified as the strongest anomaly. In addition, the second enrichment phases are located in the western and southwestern regions.

KEYWORDS

Anomaly, fuzzy AHP, geochemical exploration

I. INTRODUCTION

Selecting appropriate target areas is a crucial component of mineral exploration, significantly affecting the cost-efficiency and success of subsequent operations. In complex geochemical environments, this task requires integrating multiple, often interrelated criteria, necessitating the use of robust spatial decision-making tools.

Over the past few decades, several multi-criteria decision-making (MCDM) methods have been developed to support such tasks, including Simple Additive Weighting (SAW) (Hwang and Yoon, 1981), TOPSIS (Hwang and Yoon, 1981), Data Envelopment Analysis (DEA) (Cooper et al., 2000), and the Analytic Hierarchy Process (AHP) (Saaty, 1980). Among these, AHP has gained wide acceptance due to its ability to incorporate both quantitative data and expert judgment, making it particularly suitable for complex geoscientific decision problems.

To further enhance decision-making under uncertainty and vagueness—common characteristics of geoscientific data—fuzzy logic has been increasingly adopted. Fuzzy AHP combines the structured framework of AHP with the flexibility of fuzzy set theory, enabling better handling of imprecise or subjective information. This hybrid method has been successfully applied across various domains. In particular, the integration of fuzzy AHP with other knowledge-driven MCDM approaches has shown significant promise in mineral prospectivity mapping across a wide range of geological settings. Akinlalu et al. (2024) successfully applied fuzzy AHP for orogenic gold targeting in schist belts; Feizi et al. (2021) developed hybrid models such as FUCOM-MOORA and FUCOM-MOOSRA for greenfield exploration; and Li et al.

[1]Department of Mining Engineering, Faculty of Engineering, University of Sistan and Baluchestan, Zahedan, Iran
A.A. Daya: aliakbardaya@gmail.com

(2022) constructed a 3D prospectivity model for Fe–Cu skarn deposits.

Other studies have explored the utility of fuzzy logic and AHP-based methods in porphyry and hydrothermal vein copper systems (Zhang et al., 2017; Du et al., 2016; Pazand and Hezarkhani, 2015), as well as geospatial dataset modeling for porphyry copper prospectivity (Yazdi et al., 2015). Additionally, Mirzaei et al. (2014) demonstrated the effectiveness of fuzzy logic in iron and manganese exploration using index overlay approaches. Together, these efforts highlight the adaptability and increasing significance of fuzzy-based decision tools in exploration geoscience.

Despite these advances, many of the aforementioned studies do not systematically integrate geochemical anomaly detection—particularly methods based on fractal theory—as a foundational step prior to decision modeling. While multi-criteria decision-making tools such as fuzzy AHP provide a structured framework for prioritizing mineral targets based on geological and geochemical criteria, their effectiveness largely depends on the accuracy and quality of the input data. In mineral exploration, a critical prerequisite is the precise identification and isolation of geochemical anomalies—those zones that exhibit elevated concentrations of target elements. Without this step, the application of decision models may be compromised by background noise and statistical ambiguity.

To address this, geochemical anomaly detection methods—particularly those based on fractal theory—are employed as a foundational step in the analytical workflow. Fractal theory, introduced by Mandelbrot (1983), has been widely adopted in the geosciences for recognizing complex spatial patterns in natural systems. Cheng et al. (1994) were among the first to demonstrate that geochemical dispersion patterns often exhibit fractal characteristics, leading to the development of models aimed at differentiating true anomalies from background variations. The number–size (N–S) model, derived from Mandelbrot's original work, has become a widely accepted method for this purpose, with numerous applications in geochemical mapping and anomaly detection (Agterberg, 1995; Turcotte, 2002; Zuo et al., 2009; Wang et al., 2010).

Nevertheless, the combination of fractal-based anomaly detection with fuzzy AHP has remained underexplored, despite its potential to enhance objectivity and robustness in mineral potential modeling.

In this study, we propose an integrated two-stage approach. First, elemental anomalies of copper (Cu), gold (Au), arsenic (As), and molybdenum (Mo) are identified using the number–size fractal model. Subsequently, these delineated zones are evaluated using the fuzzy AHP method based on multiple geological, structural, and geochemical criteria. By combining fractal anomaly detection with a knowledge-driven decision model, this research addresses a methodological gap in the current literature and produces a comprehensive mineral prospectivity map for the Kamoshgaran region in northwestern Iran. This map supports more targeted and cost-effective exploration efforts.

II. NUMBER-SIZE (N-S) MODEL

The number–size (N–S) method, initially proposed by Mandelbrot in 1983, offers a valuable approach for characterizing the distribution of geochemical populations (Sadeghi et al., 2012). A key advantage of this method is that it analyzes raw geochemical data without requiring any prior preprocessing (Mao et al., 2004). Fundamentally, the N–S model elucidates the relationship between specific geochemical attributes, such as elemental concentration in the context of this study, and the cumulative number of samples exhibiting those attributes (Sadeghi et al., 2012). This relationship is often described by a power-law frequency model, which mathematically links the frequency distribution of elemental concentrations to the cumulative count of samples possessing those concentrations (e.g., Li et al., 1994; Sanderson et al., 1994; Shi and Wang, 1998; Turcotte, 1996; Zuo et al., 2009a; Sadeghi et al., 2012). The mathematical expression of this model is given by the following equation (Deng et al., 2010; Mandelbrot, 1983):

$$N(\geq \rho) = K\rho^{-D} \tag{1}$$

where ρ denotes the elemental concentration, $N(\geq\rho)$ denotes the cumulative number of samples with concentration values greater than or equal to ρ, K is constant, and D is the fractal dimension of the distribution of elemental concentrations.

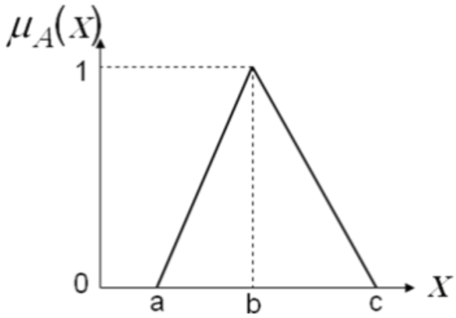
As demonstrated by Mandelbrot (1983) and Deng et al. (2010), when the cumulative number of samples (N) with concentrations greater than or equal to a certain value (ρ) is plotted against that concentration value (ρ) on a log-log scale, the resulting graph displays distinct straight-line segments. The varying slopes (-D) of these segments correspond directly to different ranges of elemental concentrations (Sadeghi et al., 2012).

III. FUZZY ANALYTIC HIERARCHY PROCESS (F-AHP)

The Analytic Hierarchy Process (AHP) provides a valuable framework for evaluating complex multi-criteria decisions that inherently involve subjective judgment. This methodology systematically structures the decision problem into a hierarchical framework, typically consisting of levels that reflect a comprehensive understanding of the situation, such as goals, criteria, sub-criteria, and alternatives. By decomposing the problem into these manageable levels, the decision-maker can focus on smaller, more targeted sets of comparisons. Within the AHP technique, elements at each level are evaluated relative to their corresponding element in the level immediately above through pairwise comparisons. While the primary objective of AHP is to capture the decision-maker's knowledge and preferences, conventional AHP has limitations in fully representing the nuances of human thought, particularly when dealing with linguistic and imprecise descriptions. To address this shortcoming, recent advancements in fuzzy decision-making have been integrated with AHP (Cheng, 1999). Fuzzy set theory, pioneered by Zadeh in 1965, provides a means of representing uncertainty within the framework of set theory. In contrast to the traditional AHP, which relies on exact numerical scales (e.g., 1–9) for scoring, many real-world decision-making scenarios involve a degree of inherent uncertainty. This uncertainty, particularly associated with translating subjective perceptions or judgments into precise numerical values, is not adequately accounted for in traditional AHP (Cheng, 1996).

Fuzzy set theory, which emulates human reasoning by utilizing approximate information and degrees of certainty to make decisions, provides a more nuanced approach for converting linguistic variables into fuzzy numbers when handling ambiguous evaluations (Zadeh, 1975). Combining fuzzy set theory with the Analytic Hierarchy Process (AHP) results in Fuzzy AHP, a method that enables a more precise representation of the decision-making process. Therefore, using fuzzy numbers and linguistic terms is more appropriate, as the traditional AHP approach can be somewhat arbitrary. A fuzzy number characterizes the relationship between an uncertain quantity x and a membership function μA(x), whose output ranges from 0 to 1. Extending classical set theory (where an element x either definitively belongs to set A or does not), a fuzzy set allows an element x to possess a degree of membership in set A, quantified by its membership function $\mu A(x)$. While fuzzy numbers can take various forms, such as bell-shaped, triangular, trapezoidal, and Gaussian, this paper adopts triangular fuzzy numbers (TFNs) to simplify implementation. A TFN is defined by three points (a, b, c) on the universe of discourse (the scale X on which a criterion is defined), representing the minimum, most likely, and maximum possible values, respectively. A visual representation of a triangular fuzzy number (TFN) is shown in Fig. 1.

This paper proposes a seven-step procedure for F-AHP, schematically illustrated in Fig. 2. These steps are then applied to a simple hierarchical structure, an example of which is shown in Fig. 3. A detailed, step-by-step description of the methodology follows.



$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a < x \le b \\ \dfrac{c-x}{c-b} & \text{if } b < x \le c \\ 0 & \text{if } x > c \end{cases}$$

Fig.1. A triangular fuzzy number

IV. CONSTRUCTION OF HIERARCHICAL STRUCTURES

The process of constructing a hierarchical model involves decomposing a complex decision problem into smaller, more manageable components organized into distinct hierarchical levels. Fig. 3 illustrates a four-level hierarchical tree. At the apex of this hierarchy is the first level, which defines the primary objective or goal. Conversely, the bottom level consists of the evaluation alternatives (options). The intermediate levels represent the various criteria and sub-criteria used for assessment.

The application of hierarchical structures in mineral exploration mapping involves organizing geological and related data according to levels of importance and scale. A hierarchical mapping strategy begins with a Regional Scale Map, which outlines major geological units, tectonic structures, and mineral districts across a broad area to identify generally prospective regions. Subsequent District Scale Maps provide a closer view of specific mineral districts, detailing geological formations, remotely sensed alteration zones, and stream sediment geochemical anomalies to pinpoint promising sub-areas. Finally, Prospect Scale Maps concentrate on specific targets within these districts, illustrating detailed outcrop geology, soil geochemistry, geophysical data, and drillhole locations to define precise drilling targets. By organizing mapping data hierarchically, explorers can efficiently progress from broad regional reconnaissance to detailed targeting, concentrating their efforts and resources on the most promising areas identified at each level.

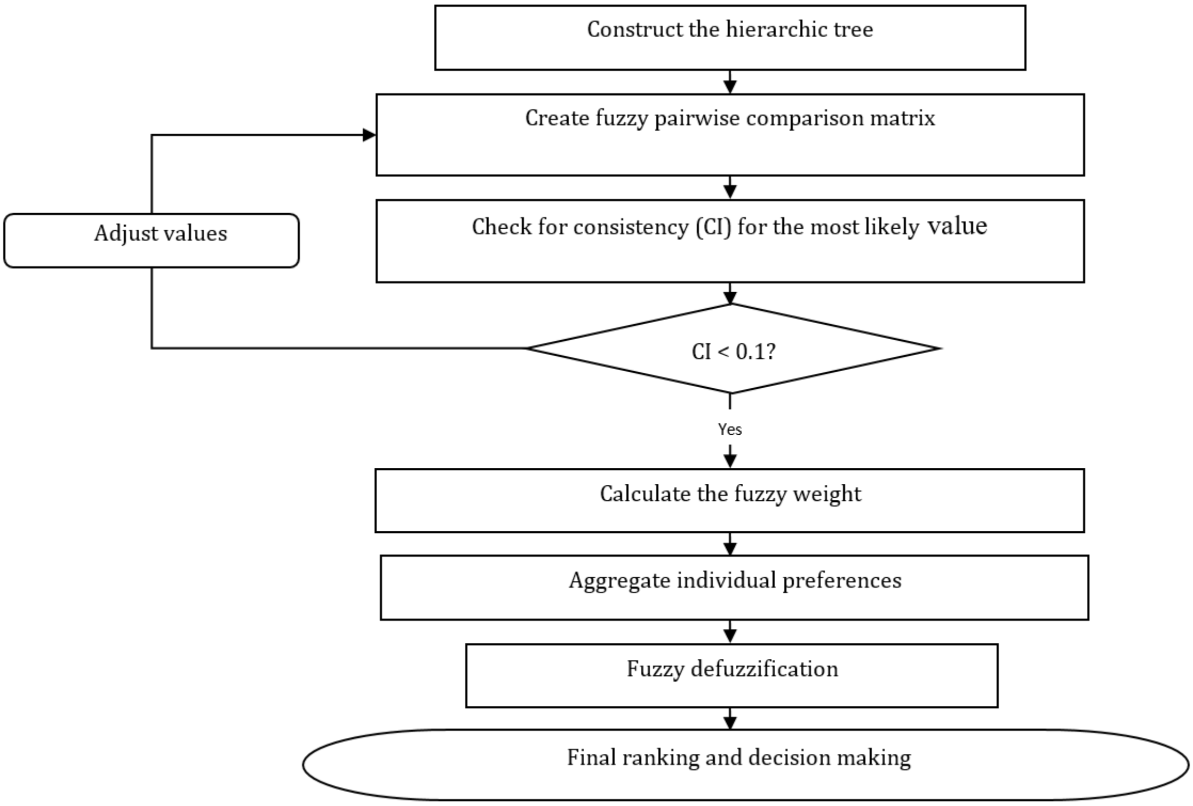


Fig. 2. Proposed methodology for fuzzy AHP

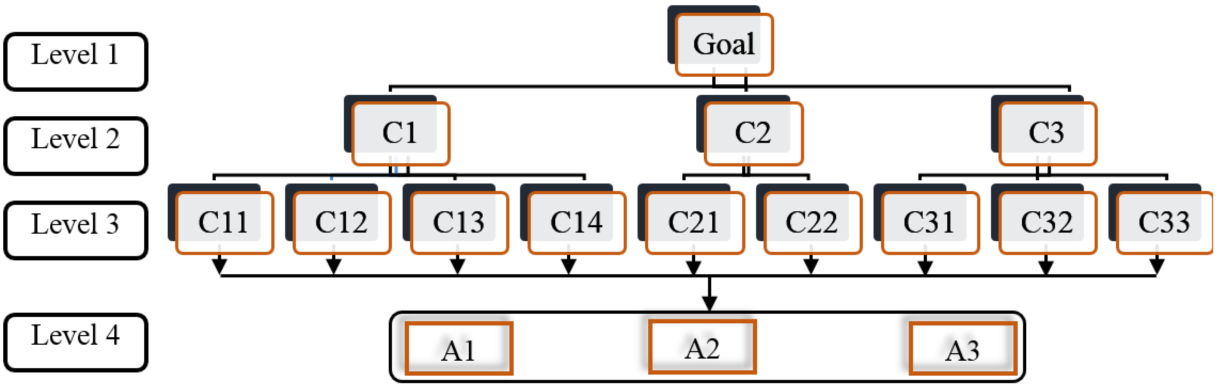


Fig. 3. Hierarchical tree example for 4-level structure

V. DEVELOPMENT OF FUZZY JUDGMENT MATRIX USING PAIRWISE COMPARISONS

In a hierarchical structure, elements at each level are compared pairwise against a specific element in the level immediately above. These pairwise comparison results in the generation of a fuzzy judgment matrix ($\tilde{J}$). The relative importance of each comparison is assigned using a scale from 1 to 9 (Saaty, 1977, 1980). These crisp values are then fuzzified, as detailed in Table 1, to account for the inherent vagueness in human perception and semantic interpretation. For n comparison items, the resulting fuzzy judgment matrix $\tilde{J}$ is represented as:

$$\tilde{J} = \begin{bmatrix} \tilde{j}_{11} & \tilde{j}_{12} & \cdots & \tilde{j}_{1n} \\ \tilde{j}_{21} & \ddots & & \tilde{j}_{2n} \\ \vdots & & \ddots & \vdots \\ \tilde{j}_{n1} & \tilde{j}_{n2} & \cdots & \tilde{j}_{nn} \end{bmatrix} \quad (1)$$

For the diagonal entries, where i=j, the value is $\tilde{j}_{ij} = 1$.

The entries in the upper right-hand triangle of the matrix represent pairwise comparisons that must be defined by the decision-maker. Conversely, the entries in the lower left-hand triangle are derived by taking the reciprocals of their corresponding upper right-hand entries, such that $\tilde{j}_{ji} = 1/\tilde{j}_{ij}$. To illustrate this, consider a pairwise comparison performed among three items (C31, C32, and C33), as shown in Fig. 3, using the relative importance values provided in Table 1. Let the level of importance of C31 to C32 is a fuzzy number $\bar{3}$; C31 to C33 is $\bar{5}$ and C32 to C33 is $\bar{4}$. Hence the judgment matrix is populated as follows:

$$\tilde{J} = \begin{bmatrix} \bar{1} & \bar{3} & \bar{5} \\ 1/\bar{3} & \bar{1} & \bar{4} \\ 1/\bar{5} & 1/\bar{4} & \bar{1} \end{bmatrix}$$

The concept of the fuzzification factor δ is introduced in Table 1. For this example, the value of the fuzzification factor δ is assumed to be 1, i.e., $\bar{3}$ meaning a TFN (2, 3, 4).

VI. Check for consistency

Consistency plays a vital role in human thinking, allowing us to organize the world based on dominance hierarchies (Saaty, 2005). This drive for coherence helps simplify complex environments and make predictable inferences (Abelson & Rosenberg, 1958). Ensuring consistency within pairwise comparisons is therefore crucial for reliable decision-making, as inconsistencies can undermine the validity and acceptance of the results (Brunswik, 1956). To quantify the degree of consistency in a pairwise comparison matrix, particularly within the framework of AHP, we calculate the Consistency Index (CI). This index indicates whether a decision-maker provides consistent judgments throughout a set of evaluations, reflecting the degree to which their preferences adhere to transitivity and overall coherence (Tversky, 1969).

$$CI = (\lambda_{max} - n)/(n - 1) \quad (2)$$

The final inconsistency in the pairwise comparisons is addressed using the consistency ratio (CR), defined as $CR = CI / RI$, where RI is the random index. The RI is obtained by averaging the consistency indices (CI) of randomly generated reciprocal matrices (Saaty, 1980). The values of RI are tabulated in Table 2. The threshold for CR is 0.1; if this threshold is exceeded, a three-step procedure should be followed (Saaty, 2005): (1) identify the most inconsistent judgment in the decision matrix, (2) determine a range of values to which the inconsistent judgment can be adjusted to reduce the associated inconsistency, and (3) ask the decision maker to reconsider the judgment to a "reasonable value." In this paper, although the pairwise comparison indices of the judgment matrix are TFNs, however, the CI and CR are evaluated using the most likely value. This approach was adopted to verify the logical coherence of expert judgments prior to fuzzy aggregation. The rationale behind this choice is that the primary role of consistency analysis in the AHP framework is to detect severe logical contradictions in the preference structure, rather than to explicitly model uncertainty. Although uncertainty is an inherent component of fuzzy AHP and is incorporated through fuzzy pairwise comparisons and propagated into the final fuzzy weights, consistency assessment is commonly performed based on the central tendency of judgments. If the modal values satisfy acceptable consistency levels, the associated fuzzy intervals generally maintain reasonable consistency behavior. Fully fuzzy consistency measures, such as those proposed by Gogus and Boucher (1998), were considered in this research. However, due to their higher computational complexity, limited methodological standardization, and relatively low adoption in applied mineral potential mapping studies, the conventional modal-value-based CI/CR evaluation was deemed more suitable for the objectives of the present case study.

Table 1. Fuzzy scales for pairwise comparisons.

Relative importance	*Fuzzy scale	Verbal judgment of preference
$\bar{1}$	(1, 1, 1)	Equal importance
$\bar{3}$	(3-δ, 3, 3+δ)	Moderate importance
$\bar{5}$	(5-δ, 5, 5+δ)	Strong importance
$\bar{7}$	(7-δ, 7, 7+δ)	Very strong importance
$\bar{9}$	(8, 9, 9)	Extreme importance
$\bar{2}, \bar{4}, \bar{6}, \bar{8}$	(x-δ, x, x+δ)	Intermediate values between adjacent scale values
$1/\bar{x}$	(1/(x+δ), 1/x, 1/(x-δ))	
$1/\bar{9}$	(1/9, 1/9, 1/8)	

*δ is a fuzzification factor.

Following the example of the judgment matrix illustrated in Step 2, the *CI* is computed. The maximum eigenvalue evaluated is ($\lambda_{\max} = 3.086$). Thus, for *n*=3, the *CI* from Eq. 2 is (*CI*=0.043) and the random index from Table 2, *RI*=0.52. Finally, the consistency ratio *CR* is calculated to be 8%.

VII. CALCULATION OF THE FUZZY WEIGHTS

Various techniques are used to compute the final fuzzy weights, such as the computation of the eigenvector (as described in Step 3), arithmetic mean, geometric mean, and others. In this paper, for ease of implementation, the geometric mean is adopted to estimate the weights. Some commonly used fuzzy arithmetic operations are listed in Table 3.

Fuzzy arithmetic operations are applied to the matrix $\tilde{J}$ to compute the fuzzy weights. Following the previous example for $\tilde{J}$, the geometric mean is computed for each row $\tilde{J}_i$. Given $\tilde{J}$ from Eq. 1, the corresponding fuzzy weights are then determined as follows::

$$\tilde{J}_i = (\tilde{j}_{i1} \otimes \cdots \otimes \tilde{j}_{in})^{\frac{1}{n}} \tag{3}$$

$$\tilde{w}_i = \tilde{J}_i \otimes (\tilde{J}_1 \oplus \cdots \oplus \tilde{J}_n)^{-1} \tag{4}$$

where $\tilde{w}_i$ is the fuzzy weight (where i =1 to n). Therefore, for C31, C32 and C33, the fuzzy weights are computed as:

$$\tilde{J}_1 = (\bar{1} \otimes \bar{3} \otimes \bar{5})^{\frac{1}{3}} =$$
$$[(1, 1, 1) \otimes (2, 3, 4) \otimes (4, 5, 6)]^{\frac{1}{3}}$$
$$= (2.0,\ 2.5,\ 2.9)$$
$$\Rightarrow \tilde{w}_1 = \tilde{J}_1 \otimes (\tilde{J}_1 \oplus \tilde{J}_2 \oplus \tilde{J}_3)^{-1}$$
$$= (0.43,\ 0.63,\ 0.89)$$

Similarly $\tilde{w}_2 = (0.19,\ 0.28,\ 0.42)$ and $\tilde{w}_3 = (0.07,\ 0.09,\ 0.14)$

The sum of the most likely values of weights $\tilde{w}_i$, i = 1, 2, 3, equals 1 (=0.63+0.28+0.09), which is the basic axiom of AHP. Therefore, crisp AHP is a special case of F-AHP, when the fuzzification factor approaches zero.

VIII. ESTABLISHMENT OF GLOBAL PREFERENCE WEIGHTS

The local priorities at each level are aggregated to obtain the final preferences for the alternatives. This computation is carried out from the evaluation of alternatives up to the top level (Goal). As shown in Fig. 3, each of the three alternatives (level 4), Ai, i = 1, 2, 3 are aggregated through level 3, level 2 and finally to level 1 (Goal). Therefore, following Fig. 3 at each level k of the hierarchical tree, the fuzzy global preference weights ($\tilde{G}_k$) are computed as:

$$\tilde{G}_k = \tilde{w}_k \cdot \tilde{G}_{k-1} \tag{5}$$

The final fuzzy AHP score ($\tilde{F}_{Ai}$) for each alternative Ai is obtained by carrying out fuzzy arithmetic summation over the global preference weights.

$$\tilde{F}_{Ai} = \sum_{k=1}^{n} \tilde{G}_k \tag{6}$$

for each alternative Ai.

IX. ORDERING THE ALTERNATIVES USING FUZZY RANKING METHODS

Defuzzification involves converting the final fuzzy AHP score $\tilde{F}_{Ai}$ into a crisp value. Once the final fuzzy AHP score ($\tilde{F}_{Ai}$) of each alternative is defuzzified, the crisp numbers are compared and ranked accordingly. In this paper, the most common centroid index method developed by Yager (1980) is employed. This index reperesent the geometric center $x_O(A_i)$ of the fuzzy number for alternative Ai. For a given TFN (a1, b1, c1) is formulated as follows:

Table 2. Random inconsistency indices (RI)

No. of criteria	1-2	3	4	5	6	7	8	9	10
RI	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Table 3. Common fuzzy arithmetical operations by two TFNs

Operators	*Formulae	Results
Summation	A + B	(a1 + b1, a2 + b2, a3 + b3)
Subtraction	A – B	(a1 – b3, a2 – b2, a3 – b1)
Multiplication	A * B	(a1 * b1, a2 * b2, a3 * b3)
Division	A/B	(a1/b3, a2/b2, a3/b1)
Scalar product	Q·B	(Q*b1, Q*b2, Q*b3)

$$x_O(A_i) = \frac{\int_0^1 A_i \mu_{Ai}(x)dx}{\int_0^1 \mu_{Ai}(x)dx} = \frac{(b_1 - a_1)(a_1 + \frac{2}{3}(b_1 - a_1)) + (c_1 - b_1)(b_1 + \frac{1}{3}(c_1 - b_1))}{(b_1 - a_1) + (c_1 - b_1)} \quad (7)$$

Where A_i is treated as a moment arm (weight function) measuring the importance of the value x. The value of may be seen as the weighted mean value of the fuzzy number A_i. Hence, the larger of the values, better the ranking of an alternative.

X. CASE STUDY

The Kamoshgaran region, located in northwestern Iran, specifically southwest of Ghorveh and east of Kurdestan province, lies within the Sanandaj-Sirjan zone (Fig. 4) and covers an area of approximately 600 km^2. This area is of significant importance due to the occurrence of gold (Au) mineralization along its fault zones. Notably, most of Iran's large and known gold deposits are concentrated within the Sanandaj–Sirjan metamorphic-structural zone, which hosts upper Paleozoic to Upper Mesozoic volcano-sedimentary sequences exhibiting lower green schist to lower amphibolite metamorphic facies (Aliyari et al., 2012).

The Sanandaj–Sirjan Zone (SSZ), a significant geological belt extending from the southwest of central Iran, lies immediately northeast of the Zagros Main Thrust. Its distinct structural and lithological characteristics indicate its formation within a deep trough separating the Arabian and Iranian Precambrian Shields, setting it apart from adjacent geological zones (Aghanabati, 2004). The consistent structural trend, uniform pattern, and prevalence of thrust faulting—hallmarks of orogenic belts in collision zones—have compellingly led prominent geologists such as Falcon (1961), Farhoudi (1978), and Alavi (1994) to classify the SSZ as a subzone within the broader Zagros Orogen. In contrast, Thiele et al. (1968) proposed that the Hercynian orogeny in this zone was associated with metamorphism. Notably, Permian rocks are typically overlain by Upper Triassic-Jurassic schists, and evidence indicates a significant Middle Triassic metamorphic event that subjected SSZ rocks to dynothermal metamorphism (Aghanabati, 2004). Synthesizing current understanding, the SSZ exhibits a primary structural evolution initiated by Late Precambrian rifting and culminating in tectonic inversion during the Early Cimmerian orogeny.

The geology of the Kamoshgaran region (Fig. 4) is characterized by a diverse suite of metamorphic rocks, including schist, marble, amphibolites, and gneiss. Interspersed within these metamorphic units are massive intrusive rocks exhibiting a range of chemical compositions. The lithological sequences in the area, as documented by Hosseiny (1999), include Triassic, Triassic-Jurassic, and Jurassic metamorphic rocks, overlain by Eocene non-metamorphic formations. Furthermore, the region hosts significant massive intrusive bodies with compositions ranging from gabbrodiorite and diorite to granodiorite and granite.

Related to economic geology, the study area contains various types of mineral deposits. Decorative stones can be classified into two primary categories: marble and igneous rocks. Additionally, the 1:10,000 map sheet indicates the occurrence of other minerals, such as pozzolan, kaolinite, iron, and titanium. However, these minerals are mainly concentrated in the northern section of the larger 100,000 sheet and are less prevalent in the southern part, which is the focus of this study. In the Kamoshgaran area, altered rocks are also present; these altered rocks are a sign of metallic mineral deposits, including gold, copper, and silver.

XI. DISCUSSION

Statistical analysis was conducted on the most important elements. The results revealed that the mean values of As, Au, Cu, and Mo were 13.55, 0.0013, 37.63, and 0.82 ppm, respectively. Table 4 lists the statistical parameters for As, Au, Cu, and Mo.

Table 4. As, Au, Cu and Mo statistical parameters of 317 stream sediment samples from Kamoshgaran Region

	As (ppm)	Au (ppb)	Cu (ppm)	Mo (ppm)
Mean	13.5516	.0013	37.6347	.8234
Median	11.6000	.0012	38.0000	.7600
Mode	18.30	.0001	38.00	.50
Std. deviation	7.32629	.00047	1.37532E 1	.35446
Variance	53.675	2.247E-7	189.151	.126
Skewness	1.423	1.955	-.138	2.238
Kurtosis	2.939	6.554	-.169	8.448
Minimum	1.66	0.007	3.00	.50
Maximum	46.90	0.043	72.50	2.91
Range	45.24	0.036	69.50	2.41

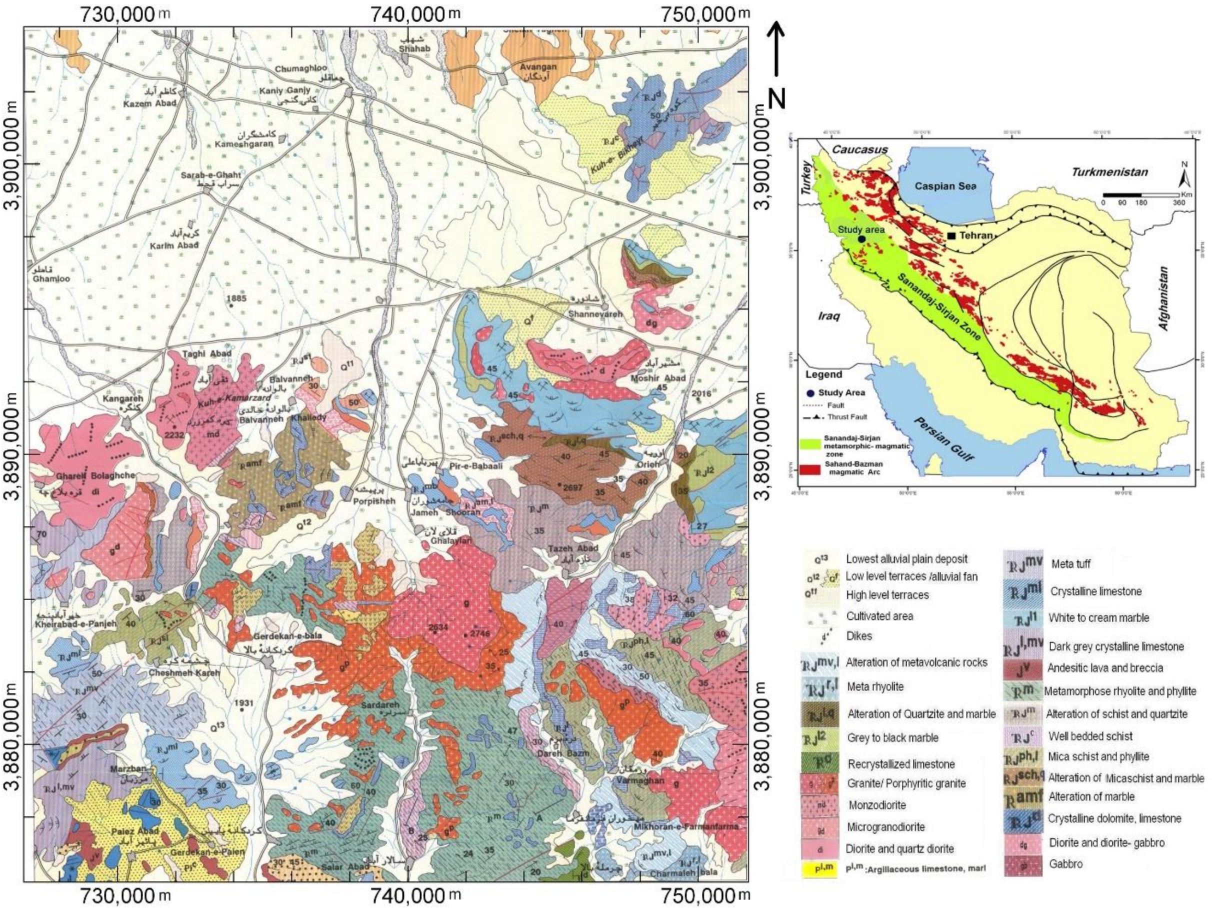


Fig. 4. A geological map of the Kamoshgaran region, adapted from the 1:100,000 Qorveh geological map. It also illustrates the regional geological context of Iran, highlighting the Kamoshgaran study area within the metamorphic–magmatic Sanandaj–Sirjan zone. This broader Iranian geology is based on a 1:2,500,000 scale map and has been compiled and simplified by Sahandi et al. (2005).

Geochemical anomalies for arsenic (As), gold (Au), copper (Cu), and molybdenum (Mo) were identified using the fractal number-size (N-S) model in 317 stream sediment samples (Fig. 5). From a geochemical viewpoint, there is a genetic relationship between As and Au mineralization. As and Cu generally exhibit a genetic association with Au. In most copper mines, Au mineralization is also present. Thus, in terms of geochemistry, these three elements may be able to provide direct mineralization information associated with the Au and other polymetallic deposits. Sampling was conducted at a density of one sample per 2 km^2 in the Kamoshgaran 1:50,000 sheet. The location map of the stream sediment samples is shown in Fig. 5. Statistical analysis carried out by Wingslib and SPSS 16 software packages. The statistical results reveal that the mean values of As, Au, Cu, and Mo are 13.5516 ppm, 0.0013 ppm, 37.6347 ppm, and 0.8234 ppm, respectively. As described in the methodology, the fractal number-size (N-S) method is a quantitative technique used in geochemistry and mineral exploration to distinguish geochemical anomalies (areas with unusually high concentrations of certain elements that may indicate mineralization) from the background levels. It is based on the principles of fractal geometry, which deals with self-similar patterns at different scales. The samples, consisting of the −80 mesh (0.18 mm) fraction, were collected from the centers of streams within the Kamoshgaran region, which was digitally represented by a grid of 10,323 cells (250 × 250 m cell size), determined by the area's geometry and sample spacing (David, 1977). Distribution models for each element were created using ordinary kriging (OK) with GSLIB software (Deutsch and Journel, 1998). To define enrichment levels, elemental concentrations were ranked from highest to lowest, and cumulative counts were calculated. Number-size (N-S) log-log plots were then generated for each element (Fig. 6).

Interpretation of these plots revealed five distinct enrichment levels for As, four for Au and Cu, and three for Mo (Fig. 6). These varying enrichment stages, as defined by the N-S method, reflect different geological controls and lithological variations.

Geochemical maps for arsenic (As), gold (Au), copper (Cu), and molybdenum (Mo) were generated using the RockWorks 15 software package. Anomalous zones for each element were delineated based on anomaly thresholds determined by the number-size (N-S) method (Fig. 6). The resulting elemental anomaly maps revealed that high arsenic concentrations were predominantly located in the eastern part of the study area, exhibiting a spatial correlation with gold anomalies (Fig. 6). Gold anomalies were primarily concentrated in the eastern and central regions, with particularly high-intensity anomalies (> 0.0024 ppm) observed in these areas. Smaller anomalous gold zones were also present in the southwestern and northern parts.

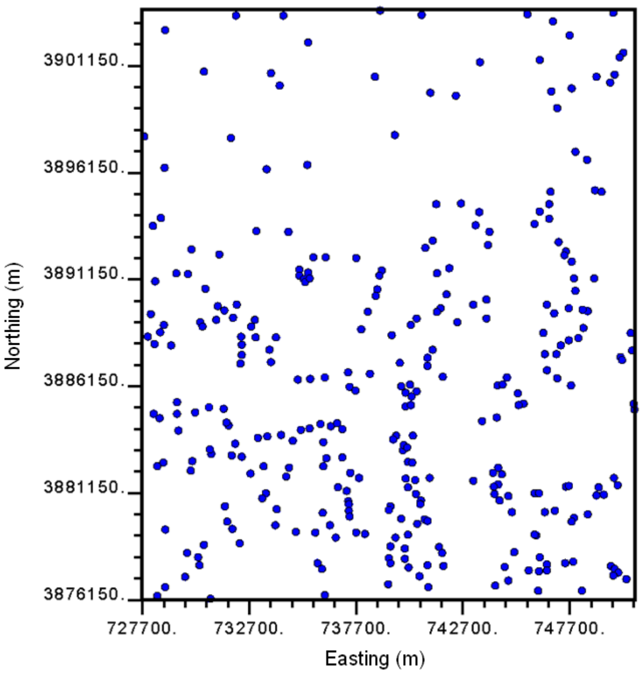


Fig. 5. Sampling locations of stream sediments from the Kamoshgaran region.

As illustrated in Fig. 6, numerous anomalous areas were identified, making the visual selection of the most promising targets challenging. To address this, a fuzzy Analytic Hierarchy Process (AHP) method was employed for objective prioritization. The main criteria for this analysis were selected based on their relevance to gold exploration and included five main criteria: lithology, fluvial sediment geochemistry, altered units, structural features, and mining index.

The fuzzy-AHP process began by establishing a hierarchical structure for the gold potential criteria. Within this hierarchy, each layer was subjected to pairwise comparisons with the elements at the level directly above it. The hierarchical structure used to identify areas prone to mineral exploration in the Kamoshgaran region is shown in Fig. 7. The hierarchical level of each criterion was determined by analyzing its relationship to gold mineralization. To assess the relative importance of these criteria, an Expert Judgment System was employed. Experts with experience in gold exploration were consulted to evaluate the relative importance of each factor. These expert opinions were then synthesized to derive a ranked order of importance for each factor, based on the nine basic terms outlined in Table 5. To enhance transparency and reproducibility, the expert judgment process is further clarified. A total of twenty experts participated in the pairwise comparison procedure. They were selected based on their professional experience in gold exploration and mineral prospectivity analysis, with backgrounds in economic geology, structural geology, geochemistry, and GIS-based mineral exploration. Each expert had more than ten years of relevant experience in gold mineralization studies or exploration projects.

Individual expert judgments were first examined for logical consistency using the consistency ratio (CR). Only matrices with CR values below the acceptable threshold ($CR < 0.1$) were considered in the analysis. This consistency check served as the primary method for validating the accuracy and reliability of the expert-provided information.

Subsequently, the individual pairwise comparison matrices were aggregated using the geometric mean to derive a single consensus matrix. This aggregation approach reduces individual subjectivity and is widely employed in both classical and fuzzy AHP applications.

This paper employs a computational technique based on the fuzzy numbers defined by Gumus (2009), as presented in Table 4. Each membership function, representing the scale of a fuzzy number, is defined by three parameters that form a symmetric triangular function: the left point, the middle point, and the right point of its range. To establish the relative importance of hierarchical elements, pairwise comparisons were conducted for all related attribute values.

Subsequently, pairwise comparison matrices were calculated for the rock subcriteria (Table 6), altered unit subcriteria (Table 7), stream sediment geochemical subcriteria (Table 8), structural data subcriteria (Table 9), mining index subcriteria (Table 10), and the main criteria.

To determine the final score, Saaty (1980) employs the hierarchical composition principle, which generates a vector representing the weighted decisions at each level of the hierarchy. Using these resulting scores, the final potential map for mineralization identifies promising areas for exploration (Fig. 8).

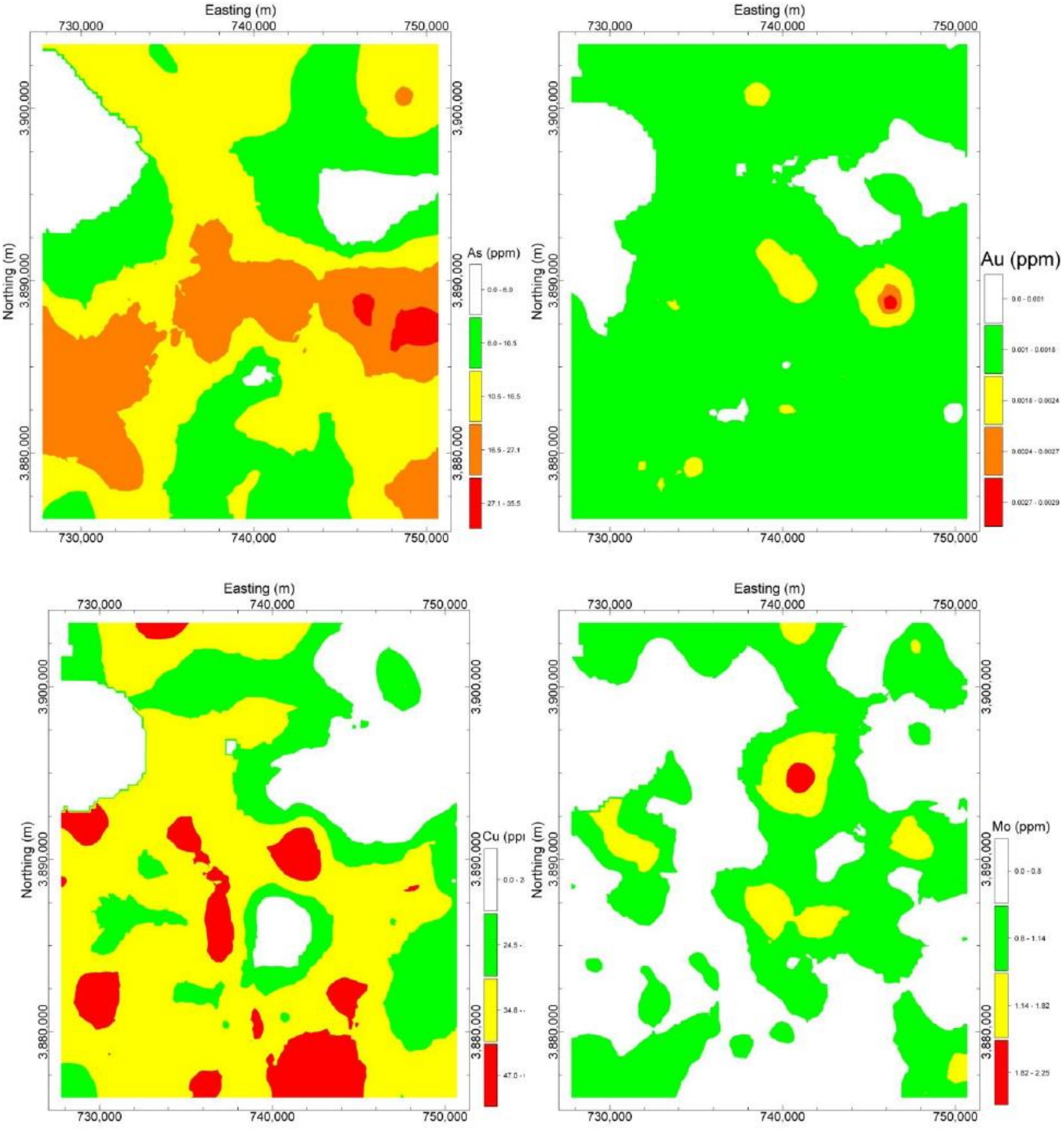


Fig. 6. N-S log–log plots of As, Au, Cu, and Mo

As shown in Fig. 8, the central and eastern parts of the region exhibit the highest ore mineralization potential; therefore, detailed exploration activities should focus on these areas. According to the fuzzy method and the integration of five important factors, their aggregate indicates that the central region is conditionally identified as the strongest anomaly. These areas warrant thorough examination. In Fig. 8, area with the highest mineralization potential are highlighted in red, while areas of secondary enrichment are shown in yellow. The second enrichment phases located in the western and southwestern regions and they are economically significant and should undergo detailed exploration studies. The areas shown in green, given that there are five small patch, have larger areas but lower importance compared to the other two regions; nonetheless, they should be examined during anomaly checking.

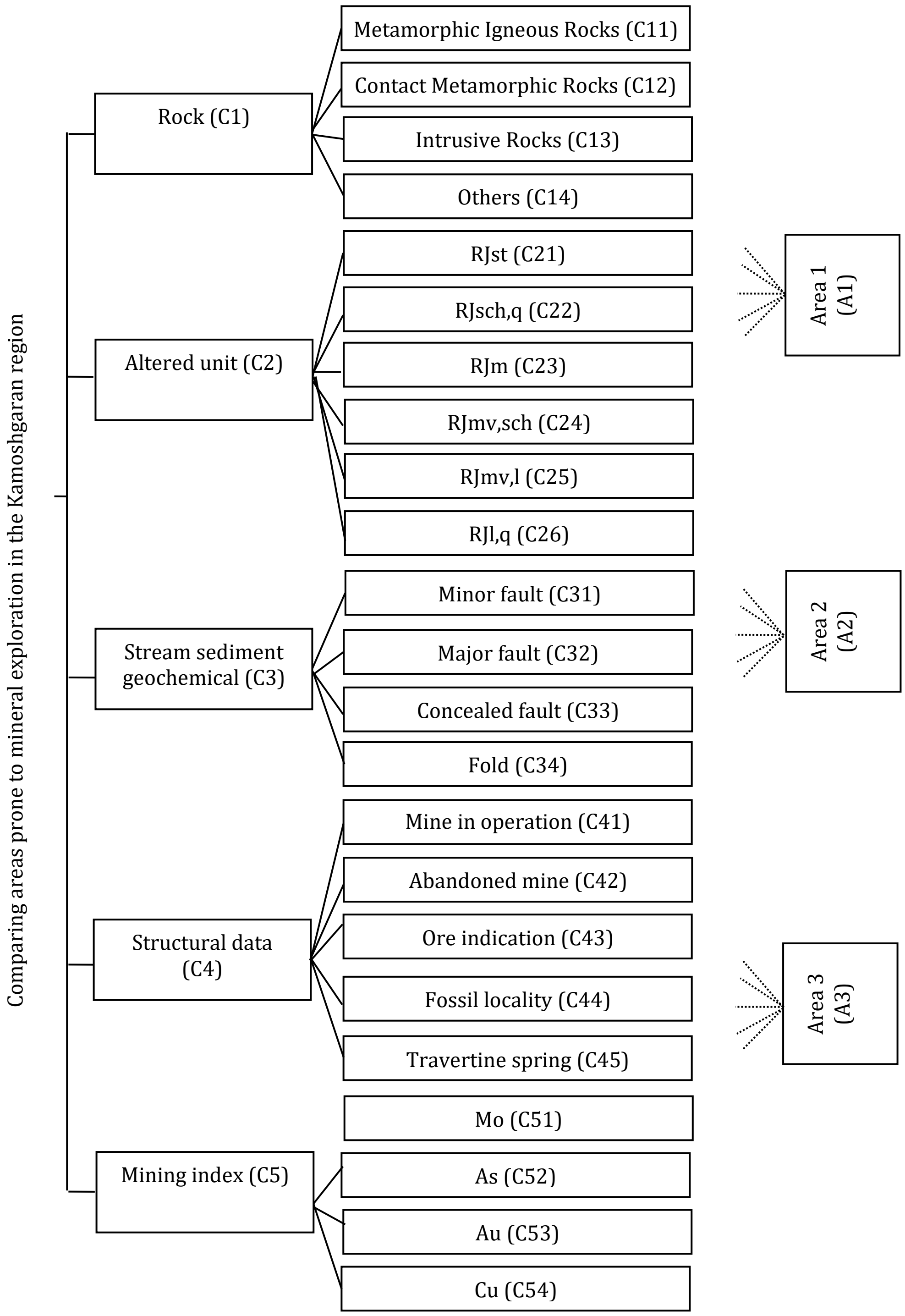


Fig. 7. Hierarchical structure for finding areas prone to mineral exploration

Table 5. Membership function of linguistic scale

Definition	Rating scale (crisp)	Fuzzy triangular	Reciprocal fuzzy
Extremely important	9	(8, 9, 9)	(1/9 1/9 1/8)
Intermediate value between 7 and 9	8	(7, 8, 9)	(1/9 1/8 1/7)
Strongly important	7	(6, 7, 8)	(1/8 1/7 1/6)
intermediate value between 5 and 7	6	(5, 6, 7)	(1/7 1/6 1/5)
Important	5	(4, 5, 6)	(1/6 1/5 1/4)
intermediate value between 3 and 5	4	(3, 4, 5)	(1/5 1/4 1/3)
Slightly important	3	(2, 3, 4)	(1/4 1/3 1/2)
intermediate value between 1 and 3	2	(1, 2, 3)	(1/3 1/2 1)
Equally important	1	(1, 1, 1)	(1 1 1)

Table 6. Pairwise comparison among rocks subcriteria

Subcriteria*	C11	C12	C13	C14
C11*	(1 1 1)	(2 3 4)	(6 7 8)	(8 9 9)
C12*	(1/4 1/3 1/2)	(1 1 1)	(2 3 4)	(5 6 7)
C13*	(1/8 1/7 1/6)	(1/4 1/3 1/2)	(1 1 1)	(3 4 5)
C14*	(1/9 1/9 1/8)	(1/5 1/6 1/7)	(1/3 1/4 1/5)	(1 1 1)

* C11= Metamorphic Igneous Rocks, C12= Contact Metamorphic Rocks, C13= Intrusive Rocks, C14= Others

Table 7. Pairwise comparison among subcriteria of altered units

Subcriteria*	C21	C22	C23	C24	C25	C26
C21	(1 1 1)	(1/8 1/7 1/6)	(1/7 1/6 1/5)	(1 1 1)	(2 3 4)	(1/8 1/7 1/6)
C22	(6 7 8)	(1 1 1)	(1 2 3)	(4 5 6)	(7 8 9)	(1 1 1)
C23	(5 6 7)	(1/3 1/2 1)	(1 1 1)	(3 4 5)	(6 7 8)	(1 1 1)
C24	(1 1 1)	(1/6 1/5 1/4)	(1/5 1/4 1/3)	(1 1 1)	(2 3 4)	(1/6 1/5 1/4)
C25	(1/4 1/3 1/5)	(1/8 1/7 1/6)	(1/8 1/7 1/6)	(1/4 1/3 1/2)	(1 1 1)	(1/9 1/9 1/8)
C26	(6 7 8)	(1 1 1)	(1 1 1)	(4 5 6)	(8 9 9)	(1 1 1)

* C21= RJst, C22= RJsch,q, C23= RJm, C24= RJmv,sch, C25= RJmv,l, C26= RJl,q

Table 8. Pairwise comparison among tectonic subcriteria

Subcriteria*	C31	C32	C33	C34
C31	(1 1 1)	(4 5 6)	(7 8 9)	(8 9 9)
C32	(1/6 1/5 1/4)	(1 1 1)	(2 3 4)	(4 5 6)
C33	(1/9 1/8 1/7)	(1/4 1/3 1/2)	(1 1 1)	(2 3 4)
C34	(1/9 1/9 1/8)	(1/6 1/5 1/4)	(1/4 1/3 1/2)	(1 1 1)

* C31= Minor fault, C32= Major fault, C33= Concealed fault, C34= Fold

Table 9. Pairwise comparison among mining index and others subcriteria

Subcriteria*	C41	C42	C43	C44	C45
C41	(1 1 1)	(3 4 5)	(4 5 6)	(8 9 9)	(8 9 9)
C42	(1/5 1/4 1/3)	(1 1 1)	(3 4 5)	(7 8 9)	(8 9 9)
C43	(1/6 1/5 1/4)	(1/5 1/4 1/3)	(1 1 1)	(3 4 5)	(5 6 7)
C44	(1/9 1/9 1/8)	(1/9 1/8 1/7)	(1/5 1/4 1/3)	(1 1 1)	(2 3 4)
C45	(1/9 1/9 1/8)	(1/9 1/9 1/8)	(1/7 1/6 1/5)	(1/4 1/3 1/2)	(1 1 1)

* C41= Mine in operation, C42= Abandoned mine, C43= Ore indication, C44= Fossil locality, C45= Travertine spring

Table 10. Pairwise comparison among geochemical subcriteria

Subcriteria*	C51	C52	C53	C54
C51	(1 1 1)	(1/6 1/5 1/4)	(1/6 1/5 1/4)	(1/6 1/5 1/4)
C52	(4 5 6)	(1 1 1)	(1/7 1/6 1/5)	(1/7 1/6 1/5)
C53	(4 5 6)	(5 6 7)	(1 1 1)	(4 5 6)
C54	(4 5 6)	(5 6 7)	(1/6 1/5 1/4)	(1 1 1)

* C51= Mo, C52= As, C53= Au, C54= Cu

XII. CONCLUSIONS

The application of the fuzzy Analytic Hierarchy Process (AHP) provided a systematic and objective framework for prioritizing key criteria relevant to mineral exploration. By integrating five critical map layers—lithology, stream sediment geochemistry, altered units, structural features, and mining index—into a hierarchical structure, the methodology enabled nuanced pairwise comparisons that accurately reflect their influence on mineralization. Leveraging expert judgment further enriched this process, ensuring that the relative importance assigned to each factor was grounded in practical experience and deep domain knowledge. The resulting ranked order offers a clear, data-driven basis for guiding exploration efforts, enhancing both the precision and reliability of identifying promising mineralization zones. Ultimately, this approach exemplifies how structured decision-making tools, combined with expert insights, can effectively address the inherent complexities of mineral exploration and facilitate more informed and successful resource discovery. The comprehensive application of the fuzzy Analytic Hierarchy Process (Fuzzy AHP) in the Kamoshgaran region has illuminated the path for future mineral exploration, identifying the central and eastern areas as prime targets due to their high ore mineralization potential. This methodological approach not only enhances the precision of mineral exploration but also optimizes resource allocation by pinpointing the most promising areas for detailed investigation, thereby increasing the efficiency and success rate of exploration activities.

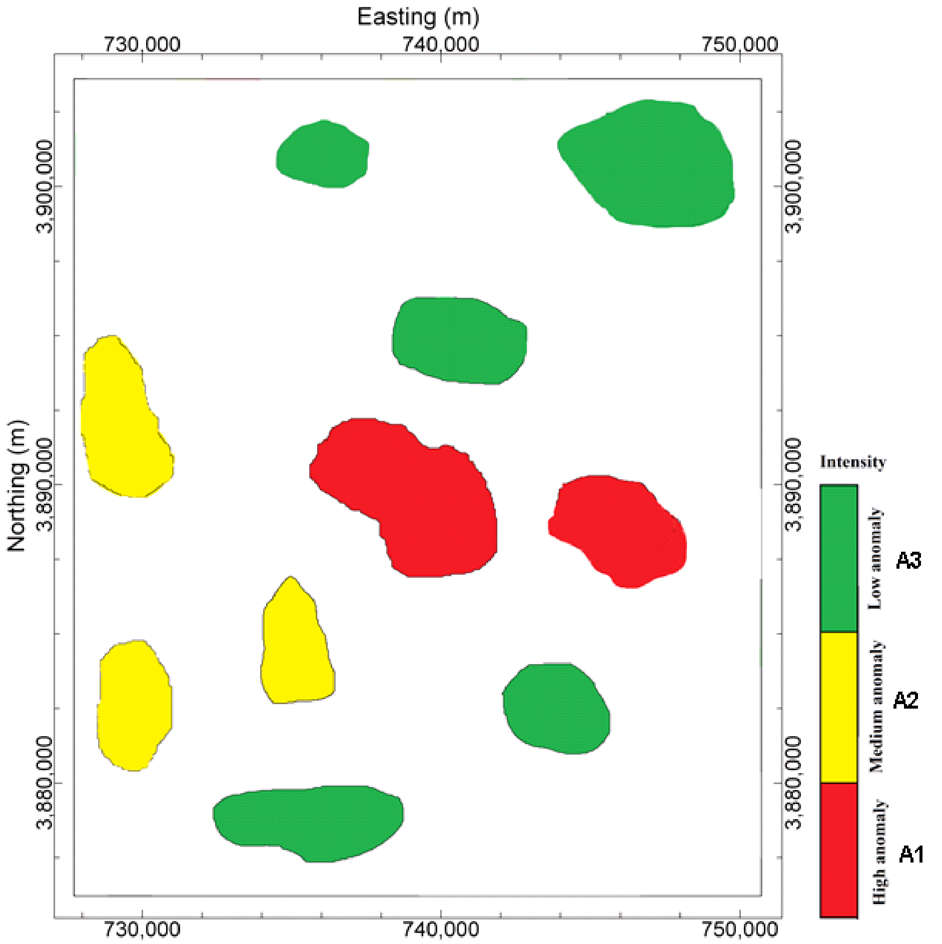


Fig. 8. The final potential map for Au mineralization, using the obtained score

REFERENCES

Abelson, R. P., & Rosenberg, M. J. (1958). Symbolic psycho-logic: A model of attitudinal cognition. Behavioral Science, 3(1), 1–13.

Aghanabati, A. (2004). Geology of Iran. Geological Survey of Iran publication.

Agterberg, F.P. (1995). Multifractal modeling of the sizes and grades of giant and supergiant deposits. International Geology Review 37, 1–8.

Alianvari, A., Katibeh, H., Sharifzadeh, M. (2012). Application of fuzzy Delphi AHP method for the estimation and classification of Ghomrud tunnel from groundwater flow hazard. Arabian Journal of Geosciences. 5:275–284. Akinlalu, A. A., Afolabi, D. O., & Sanusi, S. O. (2024). Knowledge-driven fuzzy AHP model for orogenic gold prospecting in a typical schist belt environment: a mineral system approach. Earth Systems and Environment, 8(2), 221–263.

Aliyari, F., Rastad, E., Mohajjel, M. (2012). Gold Deposits in the Sanandaj–Sirjan Zone: Orogenic Gold Deposits or Intrusion-Related Gold Systems? Resource Geology 62, 296–315.

Ataei, M., Mikaeil, R., Hosseinie, S. H., Hosseini, S. M. (2012). Fuzzy analytical hierarchy process approach for ranking the sawability of carbonate rock. International Journal of Rock Mechanics and Mining Sciences. 50:83-93.

Bejari, H., Shahriar, K., Akbarpour Shirazi, M., Khademi Hamidi, J. (2010). Optimal tunneling method selection using fuzzy multiple attribute decision making technique. ISRM International Symposium 2010 and 6th Asian Rock Mechanics Symposium - Advances in Rock Engineering, New Delhi, India. pp 240-247.

Brunswik, E. (1956). Perception and the representative design of psychological experiments. University of California Press.

Cheng, Q. (1999). Spatial and scaling modelling for geochemical anomaly separation. Journal of Geochemical Exploration 65,175–194.

Cheng, Q., Agterberg, F.P., Ballantyne, S.B. (1994). The separation of geochemical anomalies from background by fractal methods. Journal of Geochemical Exploration 51, 109–130.

Cooper, W. W., Sieford, L. M., Tone, K. (2000). Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA Solver Software, Kluwer Academic Publishers.

Dağdeviren, M., Yavuz, S., Kılınç, N. (2009). Weapon selection using the AHP and TOPSIS methods under fuzzy environment. Expert Syst Appl. 36(4):8143–8151.

David, M. (1977). Geostatistical Ore Reserve Estimation. Elsevier, Amsterdam.

Deng, J., Wang, Q., Yang, L., Wang, Y., Gong, Q., Liu, H. (2010). Delineation and explanation of geochemical anomalies using fractal models in the Heqing area, Yunnan Province, China. Journal of Geochemical Exploration 105, 95–105.

Deutsch, C.V., Journel, A.G. (1998). GSLIB: Geostatistical software library and user's guide, second ed. Oxford University Press, New York.

Du, X., Zhou, K., Cui, Y., Wang, J., Zhang, N., & Sun, W. (2016). Application of fuzzy Analytical Hierarchy Process (AHP) and Prediction-Area (PA) plot for mineral prospectivity mapping: A case study from the Dananhu metallogenic belt, Xinjiang, NW China. Arabian Journal of Geosciences, 9, 1–5.

Erensal, Y.C., Öncan, T., Demircan, M. L. (2006). Determining key capabilities in technology management using fuzzy analytic hierarchy process: a case study of Turkey. Inf Sci. 176(18):2755–2770.

Falcon, N.L. (1961). Major earth – flexturing in the Zagros Mountain of southwest Iran. Journal of the Geological Society of London 117, 367 – 376.

Farhoudi, G. (1978). A Comparison of Zagros Geology to Island Arcs. Journal of Geology 86, 325- 334.

Fattahi, H., Ebrahimi Farsangi, M. A., Shojaee, S., Mansouri, H. (2014). Selection of a suitable method for the assessment of excavation damage zone using fuzzy AHP in Aba Saleh Almahdi tunnel, Iran. Arab J Geosci. DOI: 10.1007/s12517-014-1280.

Feizi, F., Karbalaei-Ramezanali, A. A., & Farhadi, S. (2021). FUCOM-MOORA and FUCOM-MOOSRA: New MCDM-based knowledge-driven procedures for mineral potential mapping in greenfields. SN Applied Sciences, 3, 1–9.

Gogus, O., & Boucher, T. O. (1998). Strong transitivity, rationality and weak monotonicity in fuzzy pairwise comparisons. Fuzzy sets and systems, 94(1), 133-144.

Khademi Hamidi, J., Shahriar, K., Rezai, B., Rostami. J., Bejari, H. (2010). Risk assessment based selection of rock TBM for adverse geological conditions using Fuzzy-AHP. Bulletin of Engineering Geology and the Environment. 69:523-532.

Li, C., Ma, T., Shi, J. (2003). Application of a fractal method relating concentrations and distances for separation of geochemical anomalies from background. Journal of Geochemical Exploration 77, 167–175.

Li, H., Li, X., Yuan, F., Jowitt, S. M., Dou, F., Zhang, M., Li, X., Li, Y., Lan, X., Lu, S., & Ding, W. (2022). Knowledge-driven based three-dimensional prospectivity modeling of Fe–Cu skarn deposits: A case study of the Fanchang volcanic basin, Anhui province, Eastern China. Ore Geology Reviews, 149, 105065.

Mandelbrot, B.B. (1983). The Fractal Geometry of Nature. Freeman, San Francisco.

Mao, Z., Peng, S., Lai, J., Shao, Y., Yang, B. (2004). Fractal study of geochemical prospecting data in south area of Fenghuanshan copper deposit, Tongling Anhui. Journal of Earth Sciences and Environment 26, 11–14.

Önüt, S., Soner. S. (2008). Transshipment site selection using the AHP and TOPSIS approaches under fuzzy environment. Waste Manage. 28:1552–1559.

Pazand, K., Hezarkhani, A., Ghanbari, Y. (2014). Fuzzy analytical hierarchy process and GIS for predictive Cu porphyry potential mapping: a case study in Ahar–Arasbaran Zone (NW, Iran). Arabian Journal of Geosciences. 7(1):241-251.

Pazand, K., & Hezarkhani, A. (2015). Porphyry Cu potential area selection using the combine AHP-TOPSIS methods: A case study in Siahrud area (NW, Iran). Earth Science Informatics, 8, 207–220.

Rafiee, R., Ataei, M., Jalali, M.E. (2013).. The optimum support selection by using fuzzy analytical hierarchy process method for Beheshtabad water transporting tunnel in Naien, Iranian Journal of Fuzzy System. 10(6):39-51.

Saaty, T.L. (1980). The analytic hierarchy process: planning, priority setting and resource allocation. McGraw-Hill, New York.

Sadeghi, B., Moarefvand, P., Afzal, P., Yasrebi, A., Saein, L. (2012). Application of fractal models to outline mineralized zones in the Zaghia iron ore deposit, Central Iran. Journal of Geochemical Exploration 122, 9–19.

Sanderson, D.J., Roberts, S., Gumiel, P. (1994). A Fractal relationship between vein thickness and gold grade in drill core from La Codosera, Spain. Economic Geology 89, 168–173.

Shi, J., Wang, C. (1998). Fractal analysis of gold deposits in China: implication for giant deposit exploration. Earth Sciences-Journal of China University of Geosciences 23, 616–618 (In Chinese with English abstract).

Torfi, F., Zanjirani Farahani, R., Rezapour, S. (2010). Fuzzy AHP to determine the relative weights of evaluation criteria and Fuzzy TOPSIS to rank the alternatives, Applied Soft Computing. 10(2):520-528.

Turcotte, D.L. (1996). Fractals and Chaos in Geophysics, second ed. Cambridge University Press, Cambridge UK, pp. 81–99.

Turcotte, D.L. (2002). Fractals in petrology. Lithos 65, 261–271.

Tversky, A. (1969). Intransitivity of preferences. Psychological Review, 76(1), 31–48.

Yazdi, Z., Jafari Rad, A. R., & Ajayebi, K. S. (2015). Analysis and modeling of geospatial datasets for porphyry copper prospectivity mapping in Chahargonbad area, Central Iran. Arabian Journal of Geosciences, 8, 8237–8248.

Zhang, N., Zhou, K., & Du, X. (2017). Application of fuzzy logic and fuzzy AHP to mineral prospectivity mapping of porphyry and hydrothermal vein copper deposits in the Dananhu-Tousuquan island arc, Xinjiang, NW China. Journal of African Earth Sciences, 128, 84–96.