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A Review of Copula-Based Approach for Water Resources Time Series

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Abstract
This research investigates the utilization of copula functions in the water resources field, encompassing meteorological and hydrological aspects. A review of the Web of Sciences archive revealed 15143 studies featuring copula keywords. Notably, 40% of these studies pertain to copula-based simulation within this field. Groundwater studies were the least conducted, accounting for only 3% of all studies in the field. Regarding copula functions, studies were generally divided into two parts: frequency analysis and simulation, encompassing all dimensions of copula functions. Researchers confirmed the performance of copula functions in both parts. Studies in copula functions have revealed a new approach in joint frequency analysis and conditional probability estimation, based on the marginal distribution of data and their conditional density. The results indicate that in more than 2 dimensions, the tree sequence of vine copula has reduced computational complications and allows for the determination of different structures based on independent and dependent variables. Various studies have shown that the use of copula functions has been successful due to its lack of assumptions and restrictions and has good performance. For this reason, this approach is considered. The approach has been increasingly utilized in 2-dimensions and multi-variables, and continues to progress and develop.
Keywords: Conditional density, Joint CDF, Joint Probability, Simulation, Vine Copula.

1. Introduction
The frequency analysis of meteorological and hydrological events typically involves univariate determination of their return period. However, complex phenomena often have multiple characteristics. Hydrological events, for example, are often described by two or more correlation characteristics. Univariate analysis is not sufficient for flood events, which can be characterized by peak discharge, volume, and duration. These variables are not mutually independent due to the multivariate nature of the phenomenon. For systems with multiple variables, the joint return period is not equal to the return period. When estimating flood frequency, analyzing only peak discharge or flood volume frequency can lead to underestimation of risk (De Michele et al., 2005; Yue and Rasmussen, 2002).

Therefore, for a more complex hydrological phenomenon with more variables, multivariate statistical analysis is needed (Grimaldi and Serinaldi, 2008 & 2009). The most important problem of probabilistic multivariate analysis is to create a dependence structure for related random variables (Li and Zheng, 2016). Investigation the literature review shows that multivariate distribution functions have been widely used to model two or more hydrological dependent variables and their dependence structure (Salvadori and De Michele, 2007). Multivariate frequency analysis mainly includes three steps: (1) showing the importance and explaining the usefulness of the multivariate framework, (2) fitting the appropriate multivariate distribution to model the hydrological phenomenon and estimating the relevant parameters, and (3) studying the multivariate return periods or other hydrological analyzes and simulations (Chebana and Ouarda, 2011).

According to some flaws regarding multivariate distributions such as: (1): marginal distributions must be from the same family, (2): it is not clear and expressible for the case of more than two variables and (3): parameters of marginal distributions also used to model the dependence between random variables (Favre et al., 2004). Copula functions, which are the latest mathematical tools for investigating multivariate problems, were used in hydrological analysis (Salvadori et al., 2007; Xiao et al. ., 2008; Serinaldi et al., 2009). In the last two decades, copula functions have been widely used by researchers for multivariate analysis of hydroclimatological events; that all of which are expressed based on Sklar's theory (Sklar, 1959). Creating joint multivariate distributions using copulas greatly reduces the aforementioned problems.

All copula-based studies were derived from the following definition:

If the random variables $x_1,\ldots, x_n$ follow arbitrary marginal distribution functions $F_1(x_1), \ldots, F_n(x_n)$, respectively, then there exists a copula function C that combines these marginal distribution functions to form the joint distribution function $F(x_1, \ldots, x_n)$ combines as follows (Nelsen, 2006):

$$F(x_1,\ldots,x_n)=C\{(F_1(x_1),\ldots,F_n(x_n)\} \\ =C(u_1,\ldots,u_n),\ x_1,\ldots,x_n\in R \quad (1)$$

If the marginal distributions $F_i(x_i)$ are continuous, the copula function C is unique. On the contrary, if C is an n-dimensional copula function, F is an n-dimensional distribution function and $F_i(x_i),\ldots, F_n(x_n)$ is the corresponding marginal distribution. Let $C(u,v)$ be a bivariate copula function. The C function has the following characteristics (Nelsen, 2006);

For each value of u and v:

$$C(u,0)=C(0,v)=0 \quad (2)$$

$$C(u,1)=u \quad (3)$$

$$C(1,v)=v \quad (4)$$

For each u_1 and u_2, v_1 and v_2 if $u_1\le u_2$ and $v_1\le v_2$, then:

$$C(u_2,v_2)-C(u_2,v_1)- \\ C(u_1,v_2)+C(u_1,v_1)\ge 0 \quad (5)$$

So far, many families of copulas have been introduced, which mainly include the following:

(1): asymmetric elliptical copulas (normal and t); (2): Archimedes (Clayton, Gumbel, Frank, Ali-Mikhail-Haq); (3): extreme values copulas (Gumbel, Husler-Reiss, Galambos, Tawn and t-EV) and (4): other families (Plackett and Farlie-Gumbel-Morgenstern). Among the different families of copulas, Archimedean and asymmetric elliptic copulas are more popular in hydrological applications.

In the dimensions of more than 2 variables, due to the many problems and complexities in copula-based modeling, vine copula are used instead of pair-variable copula. Vine copulas allow for different types of dependencies in different pairs and also easily model higher dimensions, for example up to 10 dimensions. The starting point for constructing a multivariate distribution is to fully understand its multivariate density function and transform it into several conditional densities. Two specific characteristics of vines were identified by Bedford and Cooke (2001).

One of them is called Drawable vines (D-vine) and the other is called Canonical vines (C-vine). The R-vine copula is more flexible than the C or D type because it allows for a wider range of sequences. In order to increase sequence diversity, the R-vine copula presents a new concept. In general, a regular vine or R-vine is a:

D-vine if every node in the T-1 tree has at most two edges.

C-vine if every tree in T_i has a special node with d-1 edges. The node with d-1 edges in the T_i tree is called the root (Aas et al., 2009).

For higher dimensional distributions, there are a significant number of possible joint pair sequences. For example, there are 240 different sequences for the five-dimensional mode (Aas et al., 2009). The class of regular vine copula is very broad and includes a large number of possible copula pair decompositions. Each model provides a specific method for density decomposition. C and D vines are also a subset of regular veins. Their specification may be in the form of a set of nested trees.

Bedford and Cooke (2001) assigned the density of an n-dimensional distribution of regular copulas for D-vine and C-vine. The density $f(x_1,...,x_n)$ corresponding to the D-vine copula is written as Eq. 6. In Eq.6, j represents the trees that i runs around the edges of each tree. In the D-vine case, no node in any of the trees T_j is connected to more than two edges. In a regular vine case, each tree T_j has a unique node connected to n-j edges. Figure 3 shows a C-vine case with five variables. The n-dimensional density of a C-vine is as Eq.7.

$$\prod_{k=1}^{n} f(xk) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\ldots\ldots,i+j-1}\{F(x_i \mid x_{i+1},\ldots,x_{i+j-1}), F(x_{i+j} \mid x_{i+1},\ldots,x_{i+j-1})\} \tag{6}$$

$$\prod_{k=1}^{n} f(xk) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,j+i|1,\ldots\ldots,j-1}\{F(x_j \mid x_1,\ldots,x_{j-1}), F(x_{j+i} \mid x_1,\ldots,x_{j-1})\} \tag{7}$$

Using the C-vine copula may be useful when a particular variable is the key variable that governs the interaction in the data set. In such a situation, it may be decided to place this variable at the root of the C-vine copula.

1.2. Copula-based researches at field of drought

Analyzing and estimating the probability of occurrence of drought characteristics is one of the most widely used parameters in the field of using copula functions. This issue is of great importance due to its importance in the estimation of drought characteristics and the influence of drought characteristics on each other. In estimating the severity and duration of drought, various researches have been conducted in different regions (Salvadori and De Michele, 2004; Shiau, 2006; Serinaldi et al., 2009; Wang et al., 2010; Kao and Govindaraju, 2010; Song and Singh, 2010; Mirabbasi et al., 2012 and 2013; Zhang et al., 2015; Tsakiris et al., 2016; Chang et al., 2016; Hao et al., 2017; Abdi et al., 2017; Hangshing and Dabral, 2018; Ayantobo et al., 2019; Hui-Mean et al., 2019; Gupta et al., 2020; Zhu et al., 2020).

The most important use of copula functions in drought analysis is to estimate the probability of occurrence of drought and the return period of drought (Mirabbasi et al., 2012). The use of copula functions in drought analysis has been used in two main ways, first, the analysis of the frequency of drought characteristics and second, the analysis of a special characteristic of drought in two meteorological and hydrological fields (Nazeri Tahroudi et al., 2020). Drought in both meteorological and hydrological fields have common characteristics, which can be mentioned as time interval, severity and duration. In analyzing the frequency of drought in a univariate form, a special feature should be taken into consideration and investigated in a one-dimensional way, but considering that the features of drought are mostly affected by each other and affect each other, it is better to examined together to provide researchers with more realistic results.

Based on this, joint frequency analyzing of drought using copula functions and also estimating the return period of its characteristics using copula functions is the best method in this field. On the other hand, all the studies conducted regarding meteorological and hydrological drought and its characteristics, described the use of copula functions as satisfactory and its certainty better than the univariate mode.

1.3. Copula-based researches at field of groundwater

In many regions of the world, the quantity and quality of groundwater is strongly influenced by human activities, and its changes and pollution have become a serious problem for a society. The quantity and quality of groundwater is regularly controlled using observation wells. The use of copula functions was first introduced in the groundwater sector by Goovaerts et al. (2005) was used to model the spatial variability of arsenic concentration.

In other studies, copula functions were used in different dimensions of groundwater. One of the important issues in the use of copula functions in groundwater is the drought of groundwater, which was investigated and evaluated by various researchers (Janga Reddy and Ganguli, 2012; Saghafian and Sanginabadi, 2020; Tahroudi et al., 2020; Bai et al., 2020; Pathak and Dodamani, 2021; Sadeghfam et al., 2022; Roshni et al., 2022; Zavareh et al., 2023; Birjandi et al., 2023).

The copula-based approach has been developed due to the lack of restrictions on the application of selected distributions and the lack of restrictions on the dimensions used in different fields. The frequency analysis of groundwater level given by meteorological values as well as the river flow or the joint frequency analysis of the quality variables of the groundwater can consider the mutual influence of different parameters both on the ground surface and underground.

Due to the algebraic nature of the relationships in groundwater, the use of different models is not acceptable, and the copula-based approach is an approach that does not have this limitation, and in the field of both quantity and quality of groundwater, it can provide satisfactory results.

1.4. Copula-based researches at field of river flow and flood

One of the most important applications of copula functions is in the field of floods and flood characteristics, which has been studied by different researchers in different places. In the investigation of flood characteristics, we can refer to various studies as (Favre et al., 2004; De Michele et al., 2005; Shiau et al., 2006; Grimaldi and Serinalde, 2006a and b; Salvadori and De Michele, 2007; Genest et al., 2007; Zhang and Singh, 2007b; Kao and Govindaraju, 2008; Renard and Lang, 2007; Xiao et al., 2009; Chebana and Ouarda, 2009; Salvadori and De Michele, 2010; Latif and Mustafa, 2010; Samaniego et al., 2010; Gräler et al., 2013; Belagoune and Boutoutaou, 2013; Kuchment and Demidov, 2013; Moncoulon et al., 2013; Haberlandt and Radtke, 2014; Bezak et al., 2014; Tong et al., 2014; Sraj et al., 2015; Ming et al., 2015; Salarpour et al., 2016; Ozga-Zielinski et al., 2016; Fan et al., 2016; Candela et al., 2016; Balistrocchi and Bacchi, 2017; Xu et al., 2017; Shafaei et al., 2017; Yin et al., 2018; Ahmadi et al., 2018; Chen and Guo, 2019; Tahroudi et al., 2022; Ahangi et al., 2022; Nazeri and Mirabbasi, 2023; Nazeri Tahroudi and Mirabbasi, 2023a&b; Pronoos Sedighi et al., 2023).

Various studies on the use of copula functions in frequency analysis of extreme events such as floods show that by comparing conventional univariate models and copula-based frequency analysis, the superiority of copula functions can be seen in joint frequency analysis. Various studies in this field show that the different characteristics of extreme values are better displayed by using copula calculations due to their influence on each other, and the presented conditional return periods provide better results to researchers and decision makers.

The presentation of typical curves in field of joint frequency analysis of extreme values is also one of the other advantages of copula functions compared to the conventional mode. Also, in the discussion of joint frequency analysis with extreme values such as flood, the most widely used copula in the bivariate state is Archimedean copulas that proposed and recommended. Therefore, it can be said that copula functions are a very useful and effective tool for multivariate frequency analysis and simulation of hydrological events. The results of evaluation the literature review showed that the conditional probability of flood characterestics, when compared with the results of univariate analysis, is completely different. Therefore, when joint behavior is considered, the return periods of design floods will be different.

1.5. Copula-based researches at field of hydrometeorology

Regarding precipitation, due to the dependence of characteristics of precipitation phenomena on each other, such as intensity and duration of precipitation, the use of copula functions produces better results regarding the joint frequency analysis. De Michele and Salvadori (2003) for the first time in hydrology studies used copula functions in the analysis of precipitation frequency. Examples of the use of copula functions in multivariate analysis such as rainfall intensity and duration are as (De

Michele and Salvadori, 2003; Grimaldi and Serinaldi, 2006a; Salvadori and De Michele, 2006; Kao and Govindaraju, 2007; Zhang and Singh, 2007a; Kuhn et al., 2007; Evin and Favre, 2008; Keef et al., 2009; Serinaldi, 2009; Xu et al., 2010; Zhang et al., 2013; Ridolfi et al., 2013; Ananda, 2014; Indu and Kumar, 2014; Bárdossy and Pegram, 2014; Seo et al., 2015; Kwon and Lall, 2016; Bezak et al., 2016; Yendra et al., 2016; Guo et al., 2017; Qian et al., 2018; Suroso and Bardossy, 2018; Wei and Song, 2018 and 2019; Xiao et al., 2019; Li et al., 2019; Lazoglou and Anagnostopoulou, 2019; Nguyen-Huy et al., 2020; Dodangeh et al., 2020; Fan et al., 2020; Khashei et al., 2022; Nazeri Tahroudi et al., 2022c; Ramezani et al., 2023a).

The results of the literature reveiw showed that the use of copula functions can show the fluctuations in rainfall and as a result, the estimation of drought and its characteristics will be more realistic. Also, the results of various studies showed that two-dimensional copulas such as Clayton and Gamble-Hoggard have better performance with precipitation and its components. The important point in this section is the simulation based on the time series distribution. In fact, since the simulation is based on the time series distribution, the presented results are highly accurate.

1.6. Copula-based researches at field of simulation

In recent years, there has been a discussion about the use of copula functions in joint frequency analysis, which has led to a new understanding of their potential rapid development (Bárdossy and Pegram, 2009). This new horizon was a copula-based simulation in bivariate dimensions, rapidly developed and investigated with parameters such as drought, river flow, rainfall, rainfall-runoff, suspended sediment load, etc. (Nazeri Tahroudi et al., 2023a).

The copula-based simulation approach, using copula functions and conditional density, was first introduced as probability curves (Biller, 2009; Bárdossy and Pegram, 2009; Weiß, 2011; Oh et al., 2013; Li et al., 2013; Brechmann et al., 2013; Chen et al., 2015; Ballarin et al., 2021). Simulation based on copula functions in the form of prediction equation was first presented by Nazeri Tahroudi et al. (2020) that was presented regarding the simulation of meteorological and hydrological drought.

By using the conditional density and the provided conditional curves, they were able to provide a single diagram for each independent data regarding its simulation, and the maximum of those curves was chosen as the expected value. After that, various studies were presented regarding the simulation based on copula functions in bivariate and multivariate mode (Tahroudi et al., 2022b; Tabatabaei et al., 2022; Nazeri Tahroudi et al., 2023a&b; Vahidi et al., 2023).

1.7. Copula-based researches at field of more than two dimensions

To analyze multivariate data sets at a scale of more than 2 dimensions, flexible multivariate statistical models are needed that can adequately describe the multivariate tree structure. The copula approach allows us to seprate univariate marginals from the dependence structure. While the list of bivariate copula families is large, it is not true for the $d > 2$ case.

The motivation for vine copula models was to find a way to construct multivariate copulas using bivariate copulas as building blocks. The appropriate tool to obtain such a structure is conditioning. Joe (1996) provided the first pair in terms of distribution functions, while Bedford & Cooke (2001&2002) independently developed structures in terms of density. They also provided a framework for identifying all possible structures (Czado and Nagler, 2022). In models with high dimensions, more challenges arise for model selection.

The number of model parameters grows quadratically, which increases the computational complexity (Kurowicka, 2011; Brechmann et al., 2012, Brechmann and Joe, 2015, Joe, 2018). An alternative model class is vine copulas, which are implemented in the VineCopula package. The presentation of vine models based on the tree structure makes it possible to choose the best one and the best structure according to the relationship structures. A modified BIC measure specifically designed for high-dimensional vines was presented by Nagler et al. (2019). On the other hand, research of Müller and Czado (2018, 2019a,b) investigated the relationship between vine copulas and Gaussian directed acyclic to find scattering patterns, which leads to the presentation of dimensionality reduction techniques before using the copula model (Tagasovska et al. 2019).

Various studies in the field of using vine copula functions stated that their accuracy and performance are better than nested copulas. Also, due to the fact that the tree sequence can be checked up to the last tree, it is also possible to compare the dependence in different sequences. Vine copulas variety in choosing the tree sequence has made it possible for the user to evaluate the results based on the arrangement of the inputs. Also, in cases where the dependence between the pairs of studied series is less, vine copulas will be able to model them.

1.8. Quantitative review of studies

Nowadays, dependence modeling using copula is very common for computing high-dimensional patterns. However, using copulas in higher dimensions is challenging, where standard multivariate copulas suffer from relatively inflexible structures. Vine copulas overcome such limitations and can model complex dependence patterns by taking advantage of the rich variety of bivariate copulas as building blocks (Brechmann and Schepsmeier, 2013; Aas et al., 2009). Due to the characteristics and diversity of the tree sequence in vine copulas, nowadays its use is widely used in modeling and estimating dependencies in high dimensions (Kurowicka and Cooke, 2007; Kraus and Czado, 2017; Schepsmeier, 2019; Czado, 2019; Sun et al., 2019; Nagler et al., 2022; Nazeri Tahroudi et al, 2022b&d; Ramezani et al., 2023b; Nazeri Tahroudi and Mirabbasi, 2023c).

By checking the archive of Web of Science until the date of submission of this manuscript, 15143 articles on the topic of copula functions were observed with different keywords and copula functions. The obtained results were presented in Figure 1. According to Figure 1, it can be seen that most of the studies conducted in the field of copula functions are related to copula-based simulation with different meteorological and hydrological parameters, which accounts for 40% of the studies in the field of copula functions.

In the second place, there are copula studies in the field of drought and climate change, which accounts for 21% of the studies in this field.

The results of the analysis of copula functions in various studies in the field of water resources showed that the least study based on copulas among different parameters has been done in the field of groundwater field. Rainfall and river discharge have been ranked third and fourth in copula studies. According to the figure 1, it is possible to see the increase of studies on copula-based simulation. The copula-based approach in the simulation of various meteorological and hydrological parameters has good certainty and efficiency due to the use of the marginal distribution of observation series in the simulation and also the possibility of increasing the dimensions of the simulation with the involvement of other effective parameters (Nazeri Tahroudi and Mirabbasi, 2023b) which has caused an increase in studies in the field of simulations.

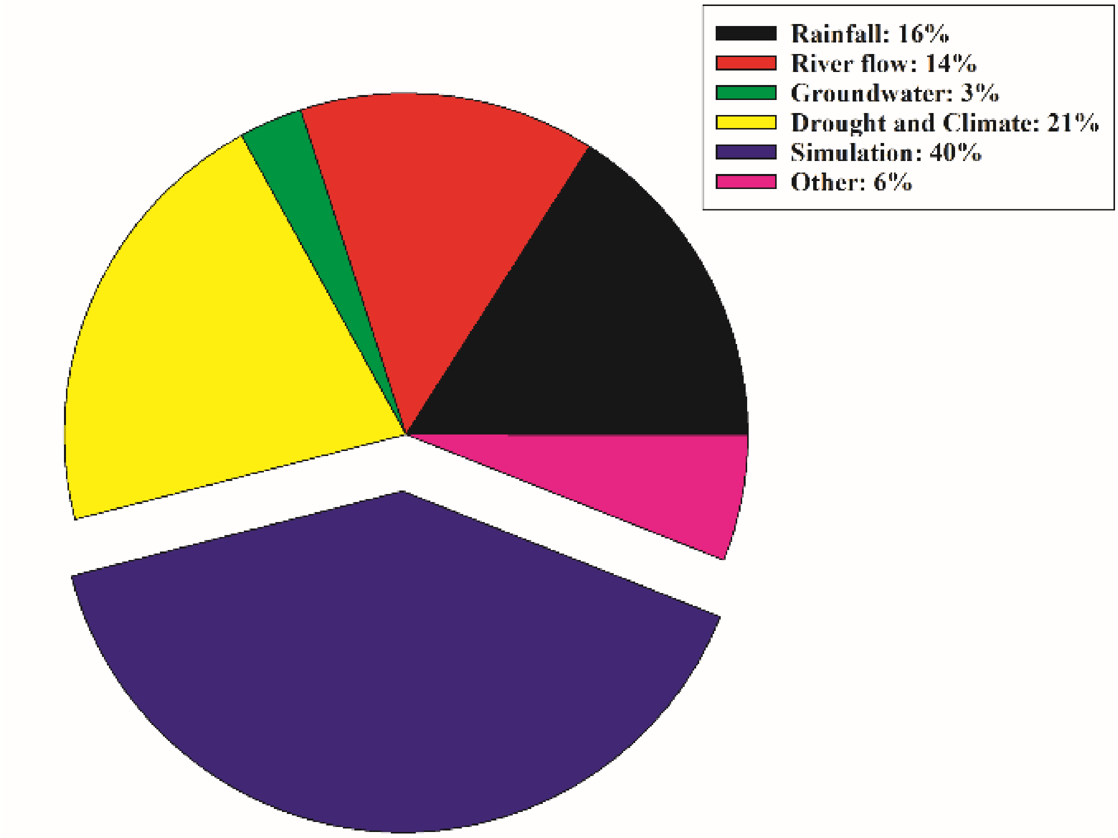

Fig. 1. The results of investigating the number of research-based on copula functions in the field of water resources

The study analyzed copula functions research in 10 subjects, including "Agricultural and Biological Sciences," "Mathematics," "Decision Sciences," "Computer Science," "Social Sciences," "Economics, Econometrics and Finance," "Engineering," "Energy," "Environmental Science," and "Earth and Planetary Sciences." The results, shown in Figure 2, demonstrate the distribution of researches across the subjects.

The results of the research conducted in the field of the use of copula functions in various subjects showed that in the field of climate change, studies based on precipitation, studies based on groundwater and studies based on river flow, the most studies of copula functions have been done in the subject of “Agricultural and Biological Sciences”, “Earth and Planetary Sciences” and “Environmental Science”. Simulation based on copula functions is also developed more in the fields of “Mathematics” and “Engineering” subjects and in the third place, “Economics, Econometrics and Finance”. In general, the research conducted in the field of copula functions can be classified as follows:

First rank: Earth and Planetary Sciences
Second rank: Agricultural and Biological Sciences
Third rank: Environmental Science
Fourth rank: Engineering
Fifth rank: Energy
6th rank: Mathematics
7th rank: Economics, Econometrics and Finance
8th rank: Decision Sciences
9th rank: Social Sciences
10th rank: Computer Science

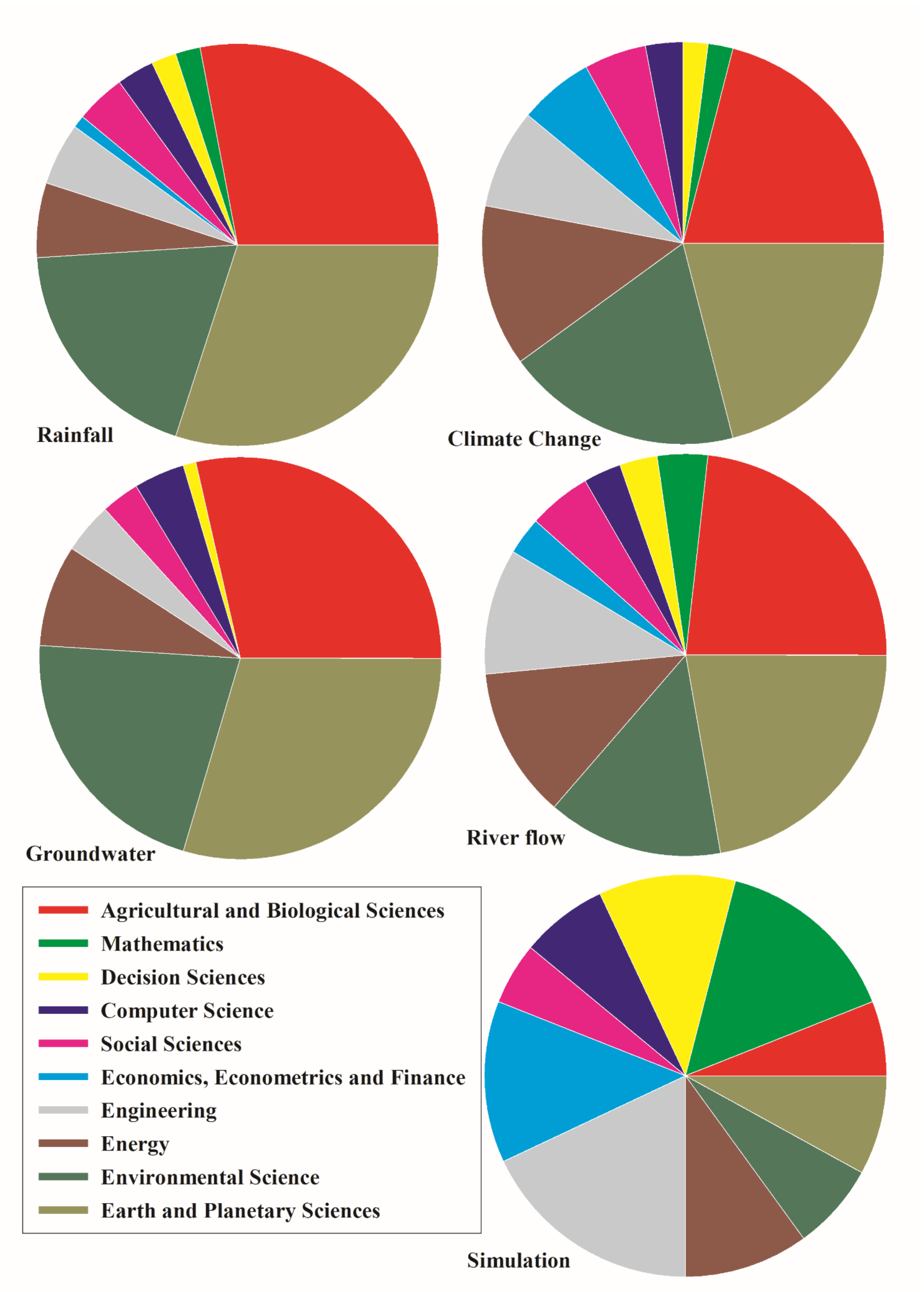

Fig. 2. The results of the research conducted in the field of copula functions in different subjects

2. Conclusion

The literature review shows the growing development of copula-based models for water resources time series. Multivariate time series models, where copulas are used to model dependence, and univariate time series are discussed. Estimation of these models is usually done parametrically, with marginal and copula distributions belonging to parametric families, or semiparametrically, where the marginal distributions are estimated nonparametrically. Nowadays, with the development of different sciences and the increase of inter-field and interdisciplinary studies, we can see the increase of efficiency of different models in all sciences. In recent decades, copulas have been introduced as an efficient tool to quantify the dependence structure between correlated quantities. The flexibility offered by copulas to create joint distributions has increased the use of this method in various studies. It is also necessary to mention that no probability distribution can exactly match the recorded data and only one of the distribution functions can be selected as the best distribution compared to other distributions. In fact, the copula is able to construct joint distributions of dependent variables with different marginal distributions. This study examines the studies conducted in the field of copula functions and its application in hydrology and water resources. The application of copula functions in different studies was investigated and the limitations and advantages of the method were evaluated. In general, from the literature review, it seems that the research conducted in the field of bivariate and multivariate copulas is often divided into two parts, 1: investigation and frequency analysis of meteorological and hydrological variables and 2: simulation based on the copula. Each of the two parts has been investigated in two and multivariate modes. Determining the return period and frequency curves of various variables are other products of copula functions that can serve as typical curves in each sub-basin. On the other hand, copula-based simulations and its combination with the conditional density of copulas can increase the performance of copula modeling. The results of the studies carried out in the field of copula functions showed that the use of conditional density and providing alternative methods with conditional return period increases the ability of copulas in conditional estimation of dependent variables on the condition of the occurrence of independent variables. In the dimensions of more than two variables, due to the computational complexity of nested copulas, we can replace vine copulas, which have a high ability in frequency analysis and copula-based simulation. The results of the conducted research showed that in the studies of the dimensions of more than two variables, vine copulas are highly accurate in frequency analysis. But these copulas have only been used in simulations, which can be developed and combined to produce a powerful tool in the field of dependent variable prediction. The results showed that in various researches, the application of copula functions in frequency analysis and copula-based simulation lacks different assumptions and also has no limitations in implementation. This approach is localized in each region and climate due to considering the marginal distribution of data.

3. Disclosure statement

No potential conflict of interest was reported by the authors.

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