Water Harvesting Research Vol. 5, No. 2, Autumn & Winter 2022, Original Paper p. 168-176

University of Birjand

10.22077/JWHR.2023.6182.1078

Frequency Analysis of Precipitation Anomaly Percentage and Stream Flow Drought Using Copula Functions

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Received: 4 March 2023/ **Revised**: 09 April 2023/ **Accepted**: 17 April 2023

Abstract

The purpose of this research is to joint frequency analyze of precipitation anomaly percentage as a meteorological drought index and flow rate at Chalekhmaz station located in the Zarinerood basin at south of Lake Urmia in the period of 1995-2016, which is based on the duration of the mentioned indicators. The results of the analysis of investigated copula functions in Zarinerood basin showed that, based on evaluation criteria, Frank's copula function describes well the dependence between two variables of the duration of anomaly percentage and the duration of hydrological drought. In Chalekhmaz station, the expectation of drought duration of 4 to 7 months for the hydrological variable and 9 to 12 months for the meteorological variable in the coming years is not far from reality. The results of the study of the return period of drought characteristics showed that in the case of the frequency of the stream flow drought index, the return period also increases with the increase in the severity of the drought. The joint frequency analysis of drought characteristics shows how meteorological and hydrological drought characteristics can be determined simultaneously in one station by using joint probabilities. This can provide users and researchers with very useful information related to the probable behavior of drought characteristics in order to optimally use of surface water. For the duration of a certain meteorological drought in a station, the duration of the hydrological drought in the hydrometric station can be determined based on the conditional probability of occurrence and also certain return periods.

Key words: Bivariate Copula, Drought, Precipitation, Return period, Trend.

1. Introduction

Floods and droughts are among the hydrological phenomena that had very profound effects on human societies and the economic losses caused by them in all corners of the world with economic development, land use change, especially in flood plains and population increase have been a growing trend (Zhang et al., 2011). Therefore, topics related to the investigation and analysis of the two characteristics of drought and flood of rivers have always been of interest. These two characteristics play a decisive role in the design of water structures, irrigation planning and the optimal use of water resources in the basins. In meteorology and hydrology, many variables have been investigated as representatives of system behavior for modeling processes, and assuming these variables independently can cast doubt on the accuracy of the final modeling results. The traditional method of multivariate analysis is to use classical multivariate distribution functions.

However, in using these functions, it is mandatory to know the marginal distributions and their types are the same, so the use of these methods faces serious limitations. According to the capabilities of the copulas, hydrological phenomena can be studied by using joint distributions or bivariate conditional distributions or higher order while maintaining the dependence structure. Since 2003, researchers such as Salvadori, Favre, and Genest have pioneered the use of copula functions in the hydrology section, so that Favre et al. (2004) used copula functions in modeling the hydrological variables of two watersheds in Quebec, Canada. They stated that the correlation between the variables in hydrology can be modeled using this method and have better results than the traditional univariate forecasting methods.

After that, the concept of copula functions was rapidly applied in various hydrological fields, including flood frequency analysis (De Michele et al., 2005; Shiau et al., 2006; Genst et al., 2007; Chebana and Ouarda, 2009; Chen et al., 2011), multivariate analysis of precipitation characteristics (Salvadori and De Michele, 2007; Kao and Govindaraju, 2008 and Zhang et al., 2013) and multivariate analysis of drought characteristics (Mirakbari et al., 2010; Mirabbasi Najafabadi et al., 2013; Zhang et al., 2015 and Abdi et al., 2017) has been investigated. In recent years, various studies have been carried out in this field (Khashei-Siuki et al., 2021; Nazeri Tahroudi et al., 2021; Tabatabaei et al., 2022; Pronoos Sedighi et al., 2022; Nazeri Tahroudi et al., 2022; Khashei-Siuki et al., 2022; Dastourani and Nazeri Tahroudi 2022; Ramezani et al., 2023; Ahangi et al., 2023). Xu et al. (2022) developed a C-vine copula-based conditional model for monthly streamflow forecasting in the Yangtze River Basin, China. They stated that the proposed model performed better results considering the highest degree of non-stationarity and has a high ability to streamflow simulation and forecasting. Sung et al. (2022) investigated the return period of flow deficit in different time using copula functions in South Korea. Different results compared to the conventional method were estimated, which helps the effective management of water resources to prepare for long-term drought risk assessment. Yang et al. (2023) proposed a bivariate joint distribution framework and a hybrid copula function to describe the properties of river flow-sediment dependent structure. They used a linear combination of Clayton, Frank and Gumbel copulas. They stated that the probability of simultaneous river flow and precipitation was 0.553. Their study showed that the proposed method can perform the accuracy of sediment statistical analysis with high accuracy. Nazeri Tahroudi and Mirabbasi (2023) developed the wavelet-copula approach in predicting the daily river flow in the Zayanderood dam basin, Iran. They compared the results of their approach with different models such as Long-Short term memory (LSTM). The results showed that the LSTM model overestimates the maximum river flow by about 30% compared to the proposed approach. For this reason, the proposed model was introduced as the best model for predicting the daily river flow in the studied sub-basin.

Considering the impact of hydrological drought on meteorological drought and precipitation anomalies, the purpose of this research is to investigate and frequency analyze of precipitation anomaly percentage on streamflow drought in Zarinerood basin using bivariate copula functions.

2. Materials and Methods

Lake Urmia, which is the center of accumulation of surface flows in excess of the consumption of all the rivers of the closed basin of Urmia, has an approximate area of 5750 square kilometers and an average height of 1276 meters above mean sea level and is located in the middle of the northern area of the basin. In this study, the meteorological and flow measurement parameters of the Zarinerood basin located in the southeast of Lake Urmia Basin were used on an annual scale and in the period of 1995-2016. This basin has an area of 7247 square kilometers and is the largest sub-basin of Lake Urmia. There are five hydrometric stations and five rain gauge stations in this basin. Zarinerood River with a length of 230 km is the longest river in Lake Urmia Basin. Figure 1 shows the location of Zarinerood sub-basin in Lake Urmia Basin.

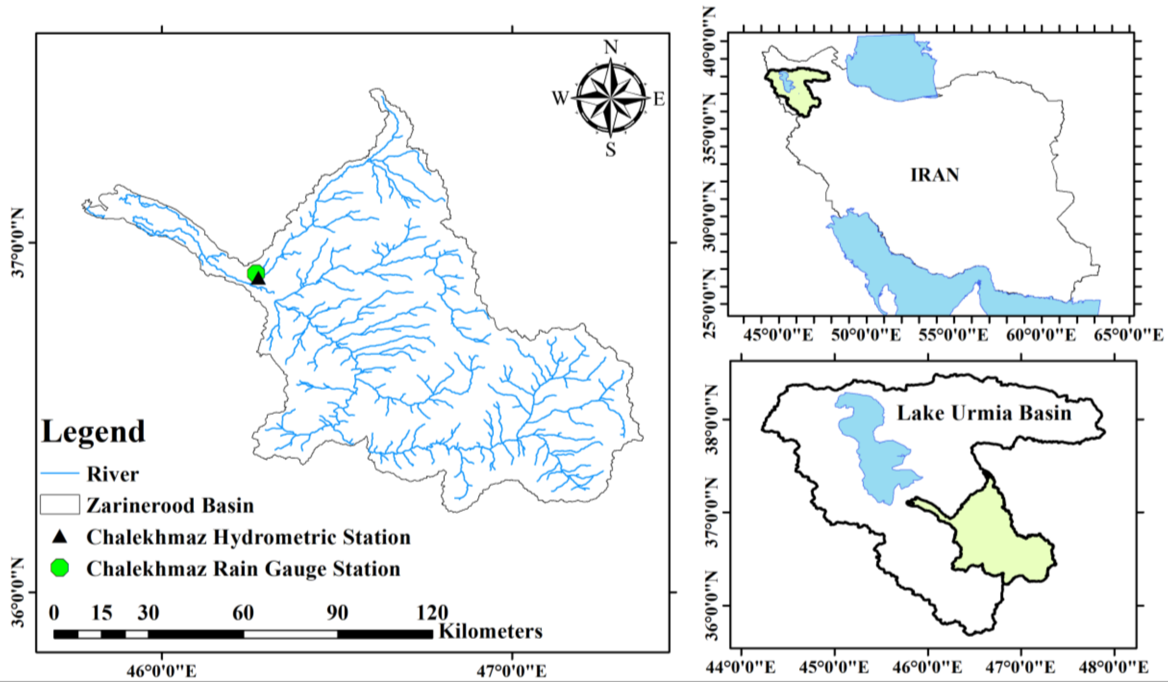

Fig. 1. The study area located in the northwest of Iran

2.1.Precipitation Anomaly Percentage (PAP)

The PAP index or precipitation anomaly percentage means the amount of annual changes in precipitation from its normal value, which is estimated as follows:

$$P_a = \frac{(p-\bar{p})}{\bar{p}} * 100 \quad (1)$$

$$\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i \quad (2)$$

In which, P_a is precipitation anomaly percentage, P is specific amount of precipitation, $\bar{p}$ is long-term average of precipitation and n is the length of the studied period.

2.2.Streamflow Drought Index (SDI)

The calculation principles of SDI are similar to SPI index. SDI is estimated using Eqs.3 to 5 (Nazeri Tahroudi et al., 2020):

$$G(x) = \frac{1}{B^{\alpha}.\Gamma(\alpha)}.x^{\alpha-1}e^{-x/B} \quad (3)$$

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1}e^{-y}dy \quad (4)$$

$$SDI_{i,k} = \frac{Q_{i,k} - \overline{Q_k}}{SD_k} \quad (5)$$

where α is the shape parameter, β is the scale parameter, x is the flow rate and $\Gamma(\alpha)$ is the gamma function. In Eq. 5, the value of $Q_{i,k}$ is equal to the flow rate of the i^{th} month in the k^{th} scale, $\overline{Q_k}$ is the average flow rate in the k^{th} scale and SD_k is the standard deviation in the k^{th} scale. The values of SDI and PAP indices are analyzed in Tables 1 and 2.

2.3.Copula theory

A copula introduction and presentation is attributed to Sklar (1959), who describes in a theory how one-dimensional distribution functions can be combined in the form of multivariate distributions.

Table 1. Classification of drought severity in the SDI index (Nalbantis and Tsakiris, 2009)

SDI	Drought classes
2 and more	Very severe wet
1.5 to 2	Moderate wet
1 to 1.5	Moderate wet
-1 to 0.99	normal
-1.5 to -1	Moderate drought
-2 to -1.5	Severe drought
-2 and less	Very severe drought

Table 2. Classification of drought severity in the PAP index (Nazeri Tahroudi et al., 2020)

PAP	Drought classes
2 and more	Very severe wet
1 to 2	Severe wet
0.5 to 2	Moderate wet
0.25 to 0.5	Low wet
0 to 0.25	Normal
-0.25 to 0	Low drought
-0.5 to -0.25	Moderate drought
-0.8 to -0.5	Severe drought
-0.8 and less	Very severe drought

For N-dimensional continuous random variables $X_1, X_2, ..., X_N$ with marginal distributions $F(x_i) = P_{x_i}(X_i \le x_i)$, the joint distribution of variable X can be defined as Eq.6.

$$H_{X_1,...,X_N}(x_1, x_2, ..., x_N) = P\left[X_1 \le x_1, X_2 \le x_2, ..., X_N \le x_N\right] \quad (6)$$

A copula is a function that connects univariate marginal distribution functions to form a bivariate or multivariate distribution function. Therefore, Sklar (1959) showed that the multivariate probability distribution H using marginal distributions and dependence structure can be expressed by the copula function C:

$$C(F_{X_1}(x_1), F_{X_2}(x_2), ..., F_{X_N}(x_N)) = H_{X_1,...,X_N}(x_1, x_2, ..., x_N) \quad (7)$$

In the above equations, $F_{X_i}(x_i)$ is ith of continuous marginal distribution and $H_{X_1,...,X_N}$ is the same cumulative distribution $X_1, X_2, ..., X_N$. Considering that for continuous random variables, the cumulative distribution function of the margins from zero to one is non-decreasing, copula C can be considered as a transformation $H_{X_1,...,X_N}$ from $(-\infty, +\infty)^N$ to $[0,1]^N$. This transformation separates the marginal distributions from each other and as a result, the copula function C is only related to the relationship between the variables and a complete description of the dependence structure is obtained. For two-dimensional copulas, the Sklar theory can be expressed as follows.

Let H be the joint distribution of X_1 and X_2 variables with cumulative distributions of $u_1 = F_{X_1}(x_1)$ and $u_2 = F_{X_1}(x_1)$. In this case, there is a two-dimensional copula function in the set of real numbers and it is expressed as equation 8.

$$H(x_1, x_2) = C(u_1, u_2) = C(F_{X_1}(x_1), F_{X_2}(x_2)) \quad (8)$$

Two-dimensional copulas have the following characteristics:

a) For each u_1 and u_2 in the interval $[0,1]$ we have:

$$C(u_1, 0) = C(0, u_2) = 0 \quad (9)$$

$$C(u_1, 1) = C(1, u_2) = 1 \quad (10)$$

It should be noted that in this study, the Inference Functions for Margins method was used to estimate the parameters of the copula function (De Michele et al., 2005). Joe (1997) recommended a two-step parametric method known as inference function for margins (IFM). The IFM method consists of two steps: first: marginal distributions are calculated from the observed values; and second: the copula dependence parameter (θ) is estimated by maximizing the log-likelihood function (Mirabbasi et al., 2012). The log-likelihood function is as follows:

$$l_\alpha = \sum_{k=1}^{n} \ln c(F_1(x_1^i; \alpha_1), ..., F_p(x_p^i; \alpha_p); \theta) + \sum_{i=1}^{n}\sum_{j=1}^{n} \ln f_j(x_j^i; \alpha_j) \quad (11)$$

First, we can obtain n separate estimates for each univariate marginal distribution, for example:

$$\bar{\alpha}_j = \arg\max \sum_{i=1}^{n} f_j(x_j^i; \alpha_j) \quad (12)$$

then the estimation of θ with previous marginal functions will be as follows:

$$\bar{\theta} = \arg\max \sum_{i=1}^{n} \ln c(F_1(x_1^i; \alpha_1), ..., F_p(x_p^i; \alpha_p); \theta) \quad (13)$$

3. Results and Discussion

As mentioned, in the first step in this study, PAP and SDI values were extracted using the monthly rainfall and river flow values of Chalekhmaz meteorological and hydrometric station in the Zarinerood Basin. The results of extracting the mentioned values were presented in Figures 2 and 3. As you can see, based on the mentioned figure, 43% of precipitation anomalies in Chalekhmaz station have experienced drought for about 64% of the years. In the case of hydrological drought, the results of the SDI values showed that in Chalekhmaz station, the river flow was dry in about 45% of the studied years. After calculating PAP and

SDI values, the duration and intensity of drought in the studied station in the period of 1994-2015 was extracted using the Run theory. Finally, after extracting the mentioned values, four new time series (severity and duration of meteorological drought and intensity and duration of hydrological drought) were obtained. Marginal distributions are fitted to the mentioned values and the results of fitting the marginal distribution functions to the mentioned series are presented in Table 3.

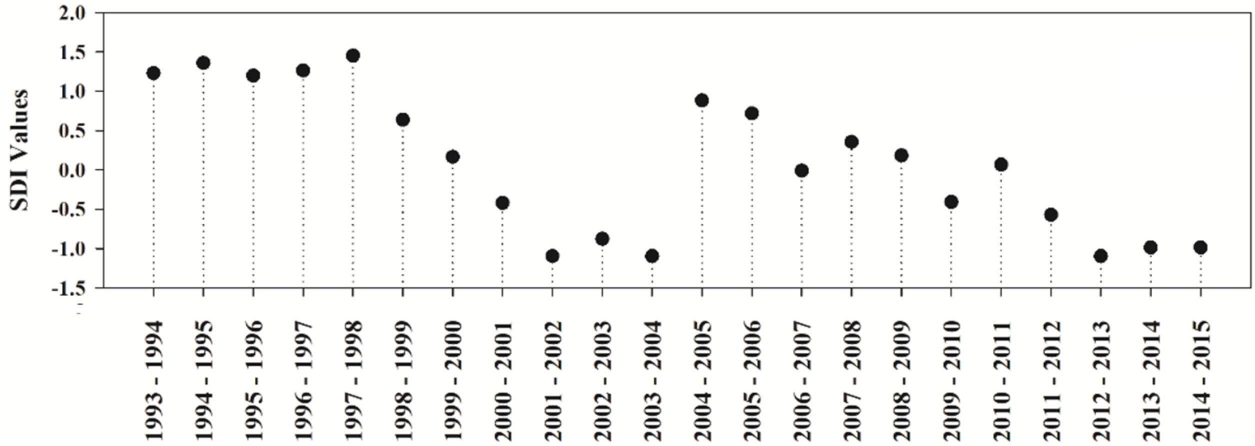

Fig. 2. The changes of SDI values in the investigated period in the Chalekhmaz station

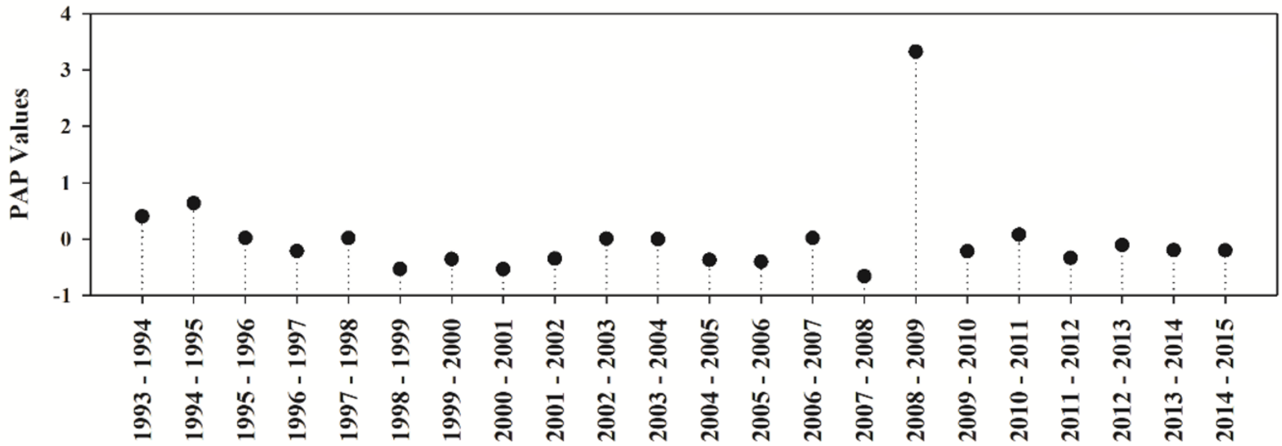

Fig. 3. The changes of PAP values in the investigated period in the Chalekhmaz station

It should be noted that the Kolmogorov-Smirnov and Anderson-Darling tests were used to select the best statistical distribution functions.

Table 3. Best statistical distributions in fitting the drought intensity and duration values

Station-Index	Drought duration	Drought intensity
Chalekhmaz-PAP	GEV	Gumbel
Chalekhmaz-SDI	GEV	Gen-Pareto

To fit the probability distributions to the investigated data, first the linear moments for the extracted series were calculated and then by equating those with the linear moments of the probability distributions, the coefficients of the desired distributions were estimated. Although observational examination of the fit of probability density functions is a common method, but the comparison between distributions that have a relatively similar fit will be difficult and accompanied by errors. After examining the marginal distribution functions and estimating its parameters, the meteorological and hydrological drought duration values were investigated using different copula functions in joint mode.

In this step, the IFM method was used to estimate the appropriate joint distribution parameters. In this regard, the seven copula functions of Ali-Mikhail-Haq (AMH), Clayton, Frank, Galambos, Gamble-Hoggard (GH), Farlie-Gumbel-Morgenstern (FGM) and Plackett were investigated and their accuracy and error rate were estimated. The accuracy and error rate of the copula functions investigated in the estimation of the return period of the meteorological and hydrological drought duration values of Chalekhmaz station and are presented in Table 4. According to the results obtained from the error statistics and the accuracy of the examined copula, it can be concluded that the Frank and Galambos copulas are the best fit with the examined data in Chalekhmaz station and have high accuracy. Also, no significant difference was found between the two functions. For this reason, Frank's copula was used in the calculations.

Table 4. The results of examining different copula functions in estimating the return period of the meteorological and hydrological drought duration values in Chalekhmaz station

Criteria	Clayton	AMH	FGM	Frank	Galambos	GH	Plackett
AIC	-4.83	-4.73	-3.13	-5.01	-5.02	-4.52	-4.94
bias	0.14	0.17	0.24	0.12	0.12	0.22	0.14
MAE	0.14	0.17	0.24	0.12	0.12	0.22	0.14
N-S	0.69	0.50	0.05	0.75	0.78	0.19	0.67
RMSE	0.16	0.20	0.27	0.14	0.13	0.25	0.16
ɵ	2.78	0.98	-0.62	13.72	6.08	1.00	20.00

Figure 4 shows the counter diagram of the joint distribution function for the investigated station. This diagram shows how to determine two durations of drought in one station at the same time using joint probabilities. This can provide users and researchers with very useful information related to the probable behavior of the drought period in order to optimally use surface water. For example, it is possible to determine the duration of the hydrological drought in the existing hydrometric station in this basin based on the conditional probability of occurrence as well as certain return periods for the duration of a specific meteorological drought in Chalekhmaz station.

Figure 5 shows the return periods of drought duration in Chalekhmaz hydrometric and meteorological station for "or" state ($X < x$ or $Y < y$). According to this figure, it can be seen that as the return period increases, the duration of drought decreases in both stations. In the case that the duration of hydrological drought in Chalekhmaz hydrometric station is between 2 and 3 months per year, the probability of occurrence increases. When the duration of meteorological drought is between 5 and 7 months per year, the return period increases. Now, by using this form, we can examine different states in joint mode.

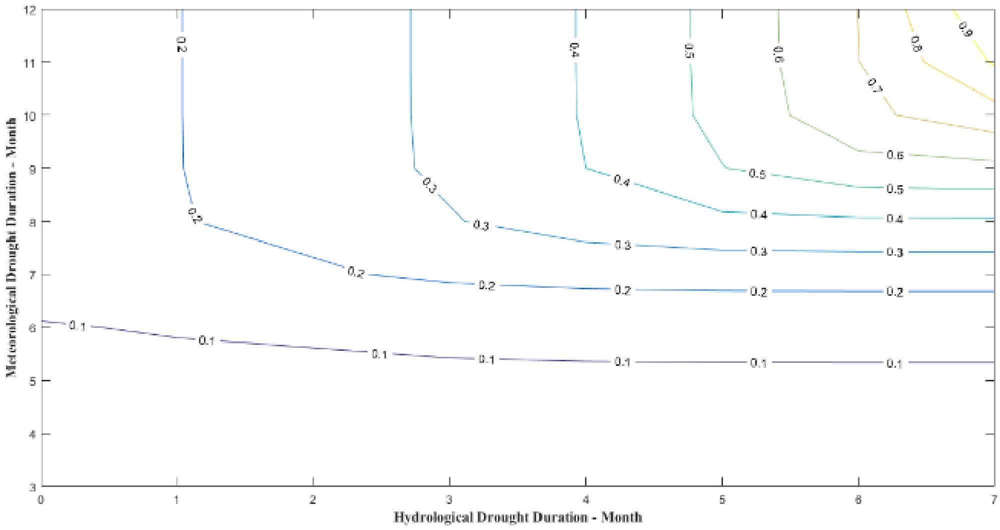

Fig. 4. Joint cumulative probability of meteorological and hydrological drought duration in Chalekhmaz meteorological and hydrometric stations

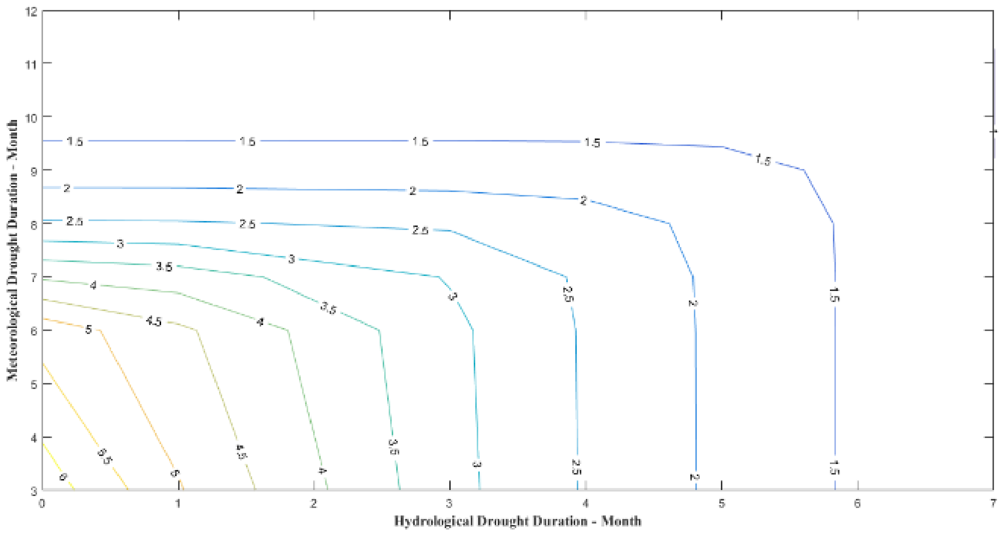

Fig. 5. The joint return period of in the "or" state for the duration of meteorological and hydrological drought in Chalekhmaz basin.

Also, this figure shows that there is a possibility of occurrence of different durations and more than 4 months for hydrological values and more than 9 months for meteorological values in each year. This is despite the fact that it is not possible to present such a comprehensive analysis in univariate analysis. Figure 6 shows the return periods of the duration of the drought in the meteorological and hydrological stations of Chalekhmaz for the "and" state. Yu and Rasmussen (2002) showed in a study that the value of the calculation return period in the two states "and" and "or" can be roughly estimated by the equations $T(X<x \text{ or } Y<y) \approx T/2$ and $T'(X<x \text{ and } Y<y) \approx T^2$ respectively. For example, for $T_x = T_y = 100$ year, the joint return period is estimated to be 50.25 years in the "OR" state and 10,000 years in the AND mode. This means that the chance of two events occurring at the same time is much less and as a result, larger return periods may occur in the period. In this way, the reason for the increase in the joint return period in Figure 6 can be justified. Now, as in the previous figure, different states can be considered and the probability of each state can be checked.

This figure also shows that the return period for the case where the duration of drought in both stations is less than a certain value varies from 20 years to 120 years and can include various combinations.

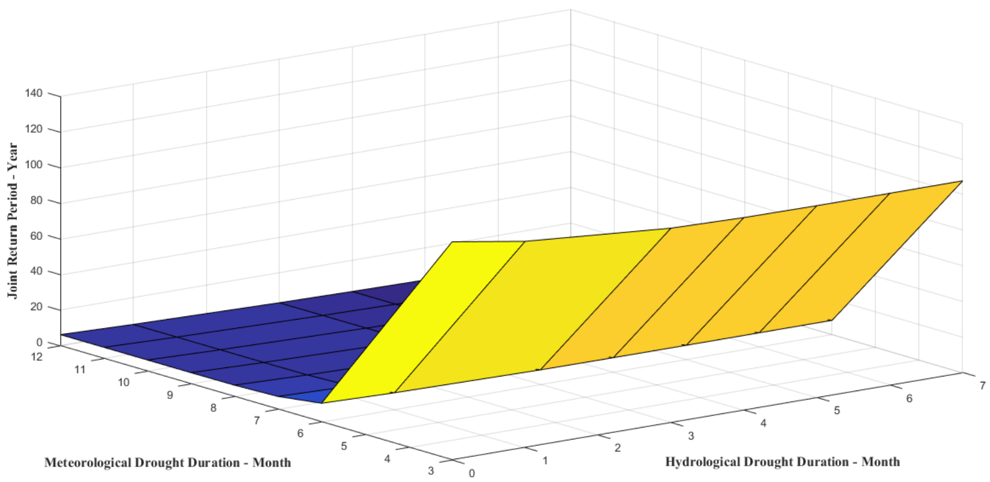

Fig. 6. The joint return period of duration of drought in Chalekhmaz station in "and" state.

This information can be very useful in the comprehensive analysis of the drought situation as well as the investigation of different climate scenarios. By using these figures, it is possible to examine different modes of joint probability. This is despite the fact that it is not possible to present such a comprehensive analysis in univariate analysis and it can be shown that the chance of two events occurring at the same time is much less and as a result, larger returns may occur in the period. In this way, the reason for increasing the return period can be justified. Also, this information can be very useful in the comprehensive analysis of the drought situation as well as the investigation of different climate scenarios.

4. Conclusion

In this study, the frequency analysis of meteorological and hydrological drought characteristics, including the duration and intensity of drought, was done in the period of 1995-2016. At first, PAP and SDI index values were extracted and meteorological and hydrological drought time series were formed. Finally, the drought duration and intensity values were extracted in the existing meteorological and hydrological stations. The results of the investigation of different statistical distribution functions showed that the trivariate GEV distribution was the suitable distribution on the investigated. Finally, with the determination of the marginal functions corresponding to the meteorological and hydrological drought duration and severity data, the corresponding copula functions were selected from among the seven investigated copula functions. The results of evaluation the error statistics and checking the accuracy of copula functions showed that among the seven examined functions, Frank's copula was the best copula function to frequency analysis of pair-variables. As mentioned, in this study, copula functions were used to perform bivariate analysis of meteorological and hydrological drought characteristics. The meaning of bivariate analysis is the simultaneous analysis of duration-duration and severity-severity of meteorological and hydrological drought. After selecting the copula function, the frequency analysis was performed considering the two states "or" and "and" for the occurrence of the desired phenomenon. The results of the bivariate analysis of meteorological and hydrological drought duration in Chalekhmaz station showed that the expected drought duration of 4 to 7 months for the hydrological variable and 9 to 12 months for the meteorological variable in the coming years is not far from reality. Also, the results of the evaluation of the joint return period of the duration-duration of the drought in this station showed that with the increase in the duration of the meteorological drought, the return period decreases in the "and" state. In the Chalekhmaz station, the results of the joint frequency analysis in the "or" state showed that the amount of meteorological drought intensity (precipitation anomaly percentage) up to -0.9 is lower than the average of the PAP index is normal and it is seen almost every year. The return period of meteorological drought intensities is more than this value with the return period of more than two years. Regarding the joint frequency analysis of the SDI index, the results showed that with the increase in the severity of the drought, the return period also increases. In the case of "and", the results of joint frequency analysis of drought intensity-intensity (meteorological and hydrological) showed that the return period also increases with the increase of drought intensity.

5. Disclosure statement

No potential conflict of interest was reported by the authors.

6. Acknowledgements

This work is based upon research founded by Iran National Science Foundation (INSF) under project No. 4005011

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