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Joint Frequency Analysis of Rainfall and Precipitation Concentration Index (PCI) at Birjand and Tabas Meteorological Stations, South Khorasan Province, Iran

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Abstract

In this study, copula functions and precipitation concentration index were used for the joint frequency analysis of the conditional probability of rainfall-rainfall and PCI-PCI at Birjand and Tabas meteorological stations in eastern Iran. Monthly rainfall data in the statistical period 1969-2018 were considered in this regard. The results of PCI at the studied stations showed that at both stations the distribution of rainfall pattern is highly irregular, which was worse at Tabas meteorological station. By selecting the appropriate marginal distribution function and also confirming the correlation between rainfall- rainfall and PCI-PCI at Tabas and Birjand meteorological stations, the Gumbel-Hougaard and Clayton copulas were selected for the pair variables, respectively. The results of conditional probability showed that with different probabilities, rainfall and PCI of each station can be estimated using the rainfall and PCI of another station. For example, according to the presented curves, if the annual rainfall of Birjand meteorological station is 220 mm, with 80% probability, the annual rainfall of Tabas meteorological station will be about 110 mm. According to the presented curves and the use of copula functions in the joint analysis of the rainfall and rainfall pattern, it is possible to better water resources management and water harvesting in the region.

Keywords: Copula Function, Distribution, Joint Probability, Rainfall Pattern.

1. Introduction

Water harvesting and water storage are of great importance due to climate change and the lack of rainfall in recent years. One of the important indices in determining the distribution and pattern of rainfall in each region is the precipitation concentration index. On the other hand, rainfall analysis, as well as its monitoring, can improve water harvesting management. The copula function developed by Sklar (1959) is a new method that replaced the univariate distribution functions. In fact, the copula can produce a join distribution of dependent variables with different marginal distribution functions. The use of multivariable analysis of hydrological parameters was presented by Snyder (1962) and Wong (1963). Subsequently, this method was used to estimate the characteristics of rainfall, flood, and drought by different researchers. The use of bivariate distributions is more widely used in the flood frequency analysis and rainfall and includes the bivariate normal distribution, bivariate exponential distribution, bivariate gamma distribution, and bivariate generalized extreme value distribution (Durrans et al, 2003; Shiau, 2003; Beersma and Buishand, 2004; Nadarajah and Gupta, 2006; He et al, 2007; Wang, 2016). Favre et al. (2004) used copula functions in modeling the hydrological variables of two basins in Quebec, Canada. They stated that the dependence between the variables in the hydrological debates can be modeled using this method and has better results than traditional univariate predictive methods. Subsequently, the concept of copula functions is rapidly investigated in various hydrological fields including flood frequency analysis (De Michele et al, 2005, Shiau et al, 2006; Genest et al, 2007; Chebana and Ourada, 2009; Chen et al, 2011; Khozeymehnezhad and Nazeri Tahroudi, 2020; Nazeri Tahroudi et al, 2021a), multivariate analysis of rainfall characteristics (Salvadori and De Michele, 2007; Zhang and Singh, 2007; Kao and Govindaraju, 2008 and Zhang et al, 2013; Tahroudi et al, 2020b; Nazeri Tahroudi et al, 2021b), multivariate analysis of drought characteristics (Salvadori and De Michele, 2004; Mirabbasi et al, 2012; Tahroudi et al, 2020a), bivariate analysis of groundwater changes (Nazeri Tahroudi et al. 2021c), simulation of potential evapotranspiration (Khashei-Siuki et al, 2021), modeling of the storm (De Michele and Salvadori, 2003) and multivariate modeling of sea storms (De Michele et al, 2007). Li et al. (2019) used the copula function to analyze the multivariate flood frequency of some rivers in the United States in the last 30 years. The results indicated that copula functions are very useful in such flood frequency analyses. Also, the results indicated that during extreme flood events, the sediment transport was reduced.

In areas where rainfall and groundwater are the most important water resources, their management and storage are very important due to climate change. Climate change in recent years has led to changes in the amount and distribution of rainfall. Multivariate analysis of rainfall and its derived indices can provide the necessary preparation for different climate conditions. The purpose of this study is the joint frequency analysis of the conditional probability of rainfall-rainfall and PCI-PCI at Birjand and Tabas meteorological stations in eastern Iran. Since rainfall is the main factor in PCI, in this study, in order to investigate the interaction of Birjand and Tabas meteorological stations with respect to each other, the joint analysis of rainfall in these two stations is evaluated using copula functions.

2. Materials and Methods

In this study, monthly rainfall data of Birjand and Tabas meteorological stations located in South Khorasan Province in eastern Iran in the statistical period of 1969-2018 have been used. South Khorasan Province in the east of Iran with an area of 95385 square kilometers is located between meridians of eastern 32′ 57˚ to 50′ 61˚ and two orbits of northern 31′ 30˚ to 15′ 45˚. Table 1 showed the characteristics of studied data. Figure 1 shows the location of South Khorasan Province and studied stations and Figure 2 showed the studied data at an annual scale in the period of 1969-2018. The distance between the two stations is about 270 km and both stations have a hot and dry climate.

Table 1- Characteristics of the studied data

Station	Minimum	Maximum	Mean	Std. deviation
Birjand	64.5	292.7	166.22	49.79
Tabas	22.5	169.6	81.31	32.30

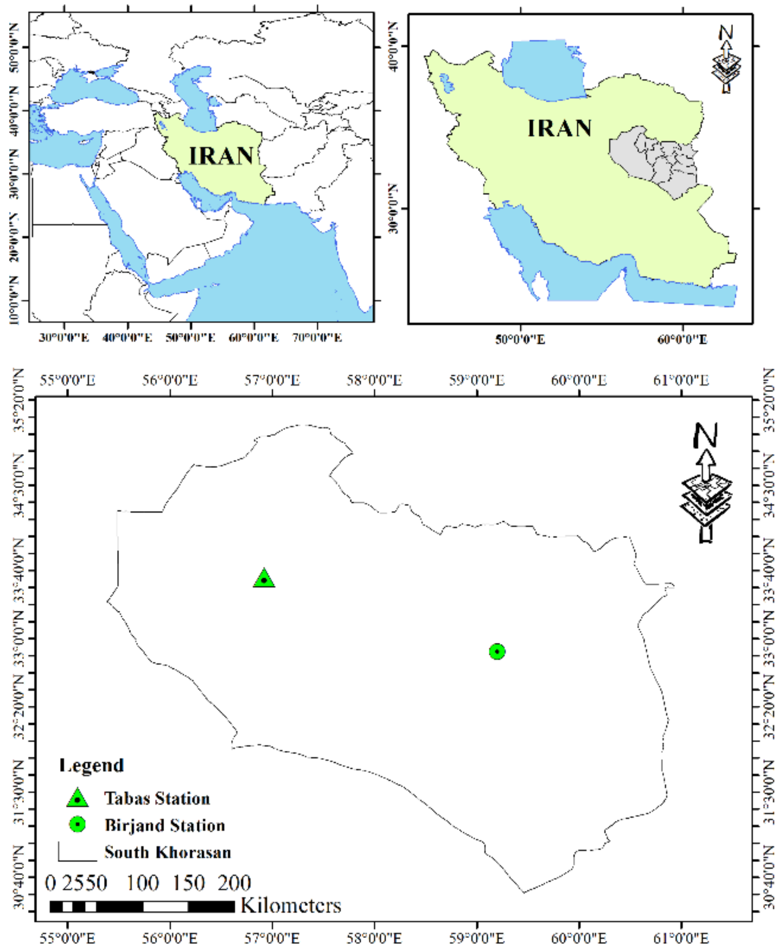

Fig. 1- Location of Tabas and Birjand meteorological stations in Iran and South Khorasan Province

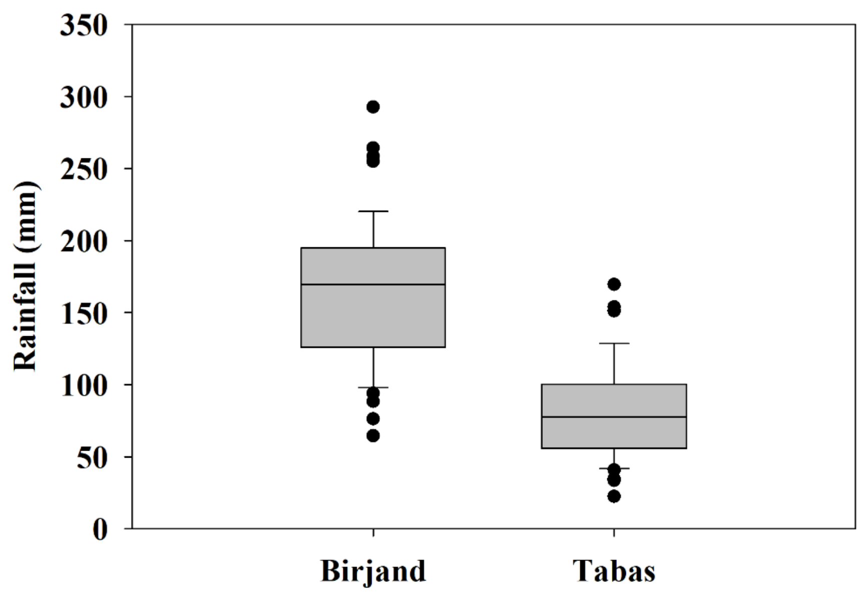

Fig. 2- Changes in rainfall at Tabas and Birjand meteorological stations at annual scale in the period of 1969-2018

2-1- Precipitation Concentration Index (PCI)

The PCI index shows rainfall variations in a region in terms of concentration and pattern of distribution. The annual scale of this index is calculated as bellow (Oliver, 1980):

$$PCI_{Annual} = \frac{\sum_{i=1}^{12} p_i^2}{(\sum_{i=1}^{12} p_i)^2} * 100 \quad (1)$$

where P_i is the amount of monthly rainfall in the i^{th} month. The PCI classification by Oliver (1980) is summarized in Table 2. The results of applying this index in the various climates of the world such as Spain, Nigeria, Iran, and India showed that the PCI index can be useful to investigate the rainfall concentration distribution (Adegun et al. 2012; Luis et al. 2011; Khozeymehnezhad and Tahroudi, 2019).

Table 2. Classification of PCI index (Oliver, 1980)

Condition	Value
uniform rainfall distribution	<10
moderate rainfall concentration	11-15
irregular distribution	16-20
strong irregularity	>20

2-2- Multivariate Frequency Analysis

Regarding the need for a more realistic statistical method, this study focuses on the use of bivariate statistical tools to joint probability analysis of rainfall and PCI. For this purpose, the method of copula functions, which has recently been used to create a dependence structure and joint probability distributions, has been utilized. The introduction and presentation of copula functions are attributed to Sklar (1959), which describes in a theory how 1-dimension distribution functions can be combined in the form of multivariate distributions. For 2-dimensional continuous random variables X_1, X_2 with marginal distribution functions $F(x_i) = P_{x_i}(X_i \leq x_i)$, the joint distribution of X variables can be defined as equation 2.

$$H_{X_1,X_2}(x_1,x_2) = P\left[X_1 \leq x_1, X_2 \leq x_2\right] \quad (2)$$

The Copula is a function that joins the univariate marginal distribution functions to form a bivariate or multivariate distribution function. Thus, Sklar (1959) showed that the probability multivariate distribution of H using the marginal distribution functions and dependence structure can be expressed by the copula function C:

$$C(F_{X_1}(x_1), F_{X_2}(x_2)) = H_{X_1,X_2}(x_1,x_2) \quad (3)$$

where, $F_{X_i}(x_i)$ is the continuous marginal distribution functions of i^{th}, and H_{X_1,X_2} is joint cumulative distribution X_1, X_2. Regarding that for continuous random variables, the cumulative marginal distribution function is non-decreasing from zero to one, the copula of C as a transform H_{X_1,X_2} from $(-\infty,+\infty)^N$ to $[0,1]^N$ can be considered. This transformation separates marginal distribution functions from each other, and consequently, the copula function of C only relates to the relationship between the variables and a complete description of the dependence structure obtained (Nelsen, 2007). For 2D copula, Sklar's theory is as follows:

Suppose H is the joint distribution of variables X_1 and X_2 with cumulative distributions $u = F_{X_1}(x_1)$ and $v = F_{X_2}(x_2)$. In this case, there exists a 2D copula function in the set of real numbers and is expressed in the form of equation (4).

$$H(x_1,x_2) = C(u,v) = C(F_{X_1}(x_1), F_{X_2}(x_2)) \quad (4)$$

For each u and v we have in $[0,1]$:

$$C(u,0) = C(0,v) = 0 \quad (5)$$

$$C(u,1) = C(1,v) = 1 \quad (6)$$

To this character, is called the boundary condition of the 2D copula function. With respect to these boundary conditions, it is concluded from equation 4 that if one of the marginal distribution functions has a value equal to zero, then the value of the copula function (joint distribution) is equal to zero. Similarly, a similar conclusion can be drawn for equation 6. For the values u_1,u_2 $(u_1 \le u_2)$ in U_1 and v_1,v_2 $(v_1 \le v_2)$ in U_2 the following non-equation is established:

$$C(u_2,v_2) - C(u_2,u_1) - C(u_1,v_2) + C(u_1,v_1) \ge 0 \quad (7)$$

The formulation of investigated copulas and the range of their dependence parameter are given in Table 3. Before using the copula functions, the dependencies between the variables used to generate the joint distribution should be examined. Kendall's tau criterion is used to determine the dependence between the two series.

Table 3. The formulation and range of the dependence parameters of the examined copulas in this study (Nelson, 2006)

Family	C (u,v)	θ
Ali-Mikhail-Haq (AMH)	$\dfrac{uv}{1-\theta(1-u)(1-v)}$	$-1 \le \theta \le 1$
Clayton	$\left(u^{-\theta}+v^{-\theta}-1\right)^{-1/\theta}$	$\theta \ge 0$
Frank	$-\dfrac{1}{\theta}\ln\left[1+\dfrac{\left(e^{-\theta u}-1\right)\left(e^{-\theta v}-1\right)}{e^{-\theta}-1}\right]$	$\theta \ne 0$
Galambos	$uv\exp\left\{\left[\left(-\ln u\right)^{-\theta}+\left(-\ln v\right)^{-\theta}\right]^{-\frac{1}{\theta}}\right\}$	$\theta \ge 0$
Gumbel-Hougaard (GH)	$\exp\left\{-\left[\left(-\ln u\right)^{\theta}+\left(-\ln v\right)^{\theta}\right]^{\frac{1}{\theta}}\right\}$	$\theta \ge 1$
Plackett	$\exp\left\{-\left[\left(-\ln u\right)^{\theta}+\left(-\ln v\right)^{\theta}\right]^{\frac{1}{\theta}}\right\}$	$\theta \ge 0$
Farlie-Gumbel-Morgenstern (FGM)	$\dfrac{1}{2}\dfrac{1}{\theta-1}\left\{1+(\theta-1)(u+v)-\left[\left(1+(\theta-1)(u+v)\right)^2-4\theta\left(\theta-1\right)uv\right]^{\frac{1}{2}}\right\}$	$-1 \le \theta \le 1$

To overcome Pearson's ρ defects, some non-parametric tests such as Kendall τ and Spearman's rank correlation (ρ) have been considered. Also, for more information about copula function and joint probability function see mentioned references (Ramezani et al, 2019; Mirabbasi et al, 2012; Nelsen, 2006; De Michele et al, 2005). Also, to evaluate the copula calculation, Akaike Information Criterion (AIC), Mean Absolute Error (MAE), Nash-Sutcliffe Efficiency (NSE), and Root Mean Square Error (RMSE) were used. For more information about these statistics, see (Nash and Sutcliffe, 1970; Akbarpour et al, 2020).

3. Results and Discussion

As mentioned, in this study, a copula theory was used for joint frequency analysis of annual rainfall at Birjand meteorological station-annual rainfall at Tabas meteorological station and annual PCI at Birjand meteorological station-annual PCI at Tabas meteorological station. First, the correlation of the studied pair variables was examined using Kendall’s tau. Proper correlation is a prerequisite for copula functions. The results of the correlation of mentioned pair variables are presented in Figure 2. In this figure, the main diameter is a histogram of the data, the upper triangular elements are Kendall’s tau and the lower triangular elements are the data scattering.

After confirming the correlation of the studied pair variables using Kendall’s tau presented in Figure 3, the appropriate marginal distribution function to the annual rainfall data at Tabas and Birjand meteorological stations as well as PCI in these two stations were selected. According to the study of different distribution functions, the GEV distribution was selected as the best distribution for the rainfall data at Tabas meteorological station and the PCI at Tabas and Birjand meteorological stations. Regarding the annual rainfall data at Birjand meteorological station, the Wakeby distribution had a good fit with the data and was selected as the best marginal distribution function. Using the selected marginal distribution functions, different copula functions were investigated and the best copula was selected based on AIC, MAE, NSE, and RMSE. It should be noted that the selected evaluation criteria are based on the differences between the studied copulas and the empirical copulas. The results of the evaluation of different copula functions regarding the pair variables of rainfall (annual rainfall at Tabas and Birjand meteorological stations) and the pair variables of PCI (annual PCI at Tabas and Birjand meteorological stations) were presented in Tables 3 and 4, respectively. In this section, 7 common copula functions in hydrology and water resources, including Clayton (Cly), Ali-Mikhail-Haq (AMH), Farlie-Gumbel-Morgenstern (FGM), Frank (Fra), Galambos (Gal), Gumbel-Hougaard (GH), and Plackett (Pla) were fitted to the mentioned pair variables and the best copula was selected based on evaluation statistics.

The last column of both tables (3 and 4) shows the θ values corresponding to the studied copulas.

Table 3 showed the results of the study of different copula functions in analyzing the frequency of pair variables of rainfall at Birjand and Tabas meteorological stations in the statistical period 1969-2018, at an annual scale. Regarding Table 3, the results showed that other than Farlie-Gumbel-Morgenstern and Ali-Mikhail-Haq copulas, other copulas have NSE of more than 90%. These conditions are the same for the PCI pair variables according to Table 4. According to Tables 3 and 4, it can be seen that the accuracy and performance of the Clayton, Frank, Galambos, and Gumbel-Hougaard copulas are almost similar. Finally, according to different evaluation statistics, Gumbel-Hougaard and Clayton copulas were selected for rainfall and PCI pair variables, respectively. Considering the selected copula functions and their coefficients, while examining and using the conditional density of the copula functions, the joint probability of occurrence of the studied pair variables was presented in Figures 4 and 5. Figure 4 showed the joint cumulative probability of rainfall and Figure 5 showed the joint cumulative probability of PCI at Birjand and Tabas meteorological stations.

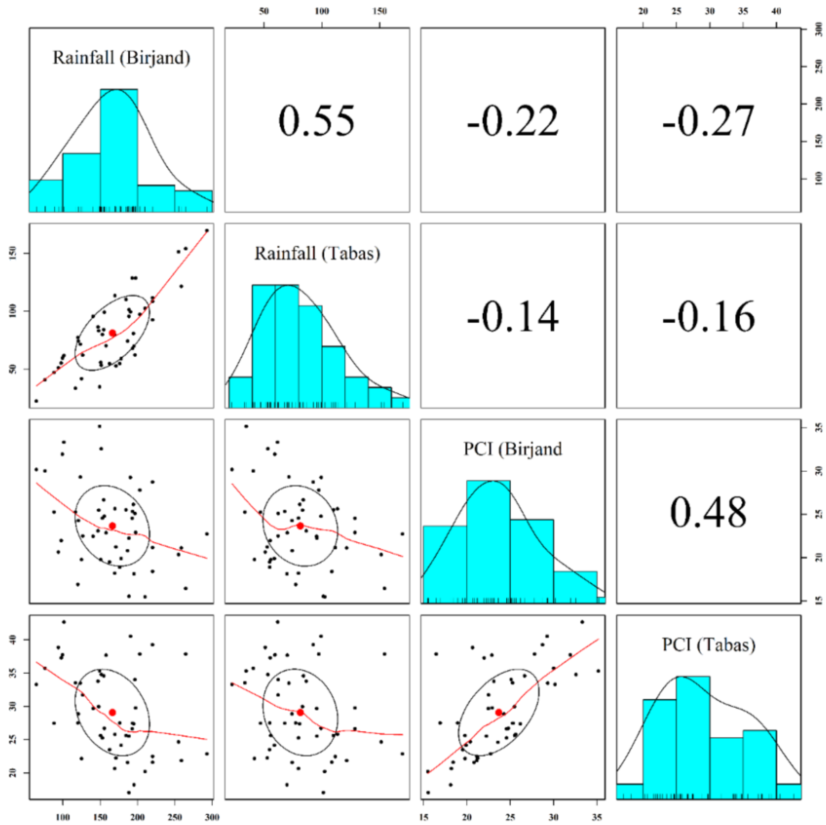

Fig. 3- Histogram, correlation, and scattering of studied pair variables

Table 3- Results of evaluation of the different copula functions in joint frequency analysis of rainfall at Birjand and Tabas meteorological stations

Copula	AIC	MAE	NSE	RMSE	θ
Clayton	-8.70	0.04	0.97	0.05	11.31
Ali-Mikhail-Haq	-8.52	0.12	0.78	0.14	1.00
Farlie-Gumbel-Morgenstern	-8.13	0.21	0.38	0.23	-0.93
Frank	-8.71	0.05	0.97	0.05	20.00
Galambos	-8.74	0.04	0.98	0.04	8.07
Gumbel-Hougaard	**-8.74**	**0.04**	**0.98**	**0.04**	**8.79**
Plackett	-8.63	0.08	0.91	0.08	20.00

Table 4- Results of evaluation of the different copula functions in joint frequency analysis of PCI at Birjand and Tabas meteorological stations

Copula	AIC	MAE	NSE	RMSE	θ
Clayton	**-7.92**	**0.04**	**0.97**	**0.05**	**9.39**
Ali-Mikhail-Haq	-7.63	0.12	0.78	0.14	1.00
Farlie-Gumbel-Morgenstern	-7.02	0.19	0.48	0.21	-0.41
Frank	-7.89	0.05	0.96	0.06	20.00
Galambos	-7.89	0.05	0.96	0.06	6.23
Gumbel-Hougaard	-7.89	0.05	0.96	0.06	6.93
Plackett	-7.79	0.08	0.90	0.09	20.00

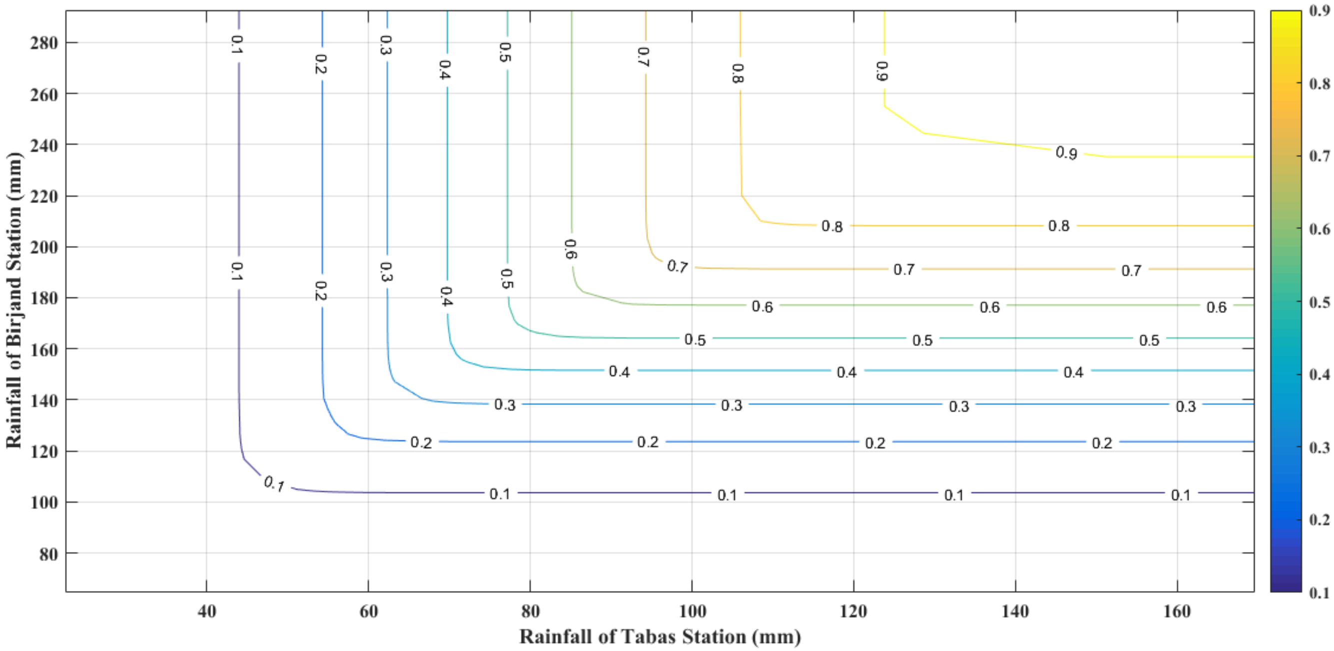

Fig. 4. Joint cumulative probability of rainfall at Birjand and Tabas meteorological stations

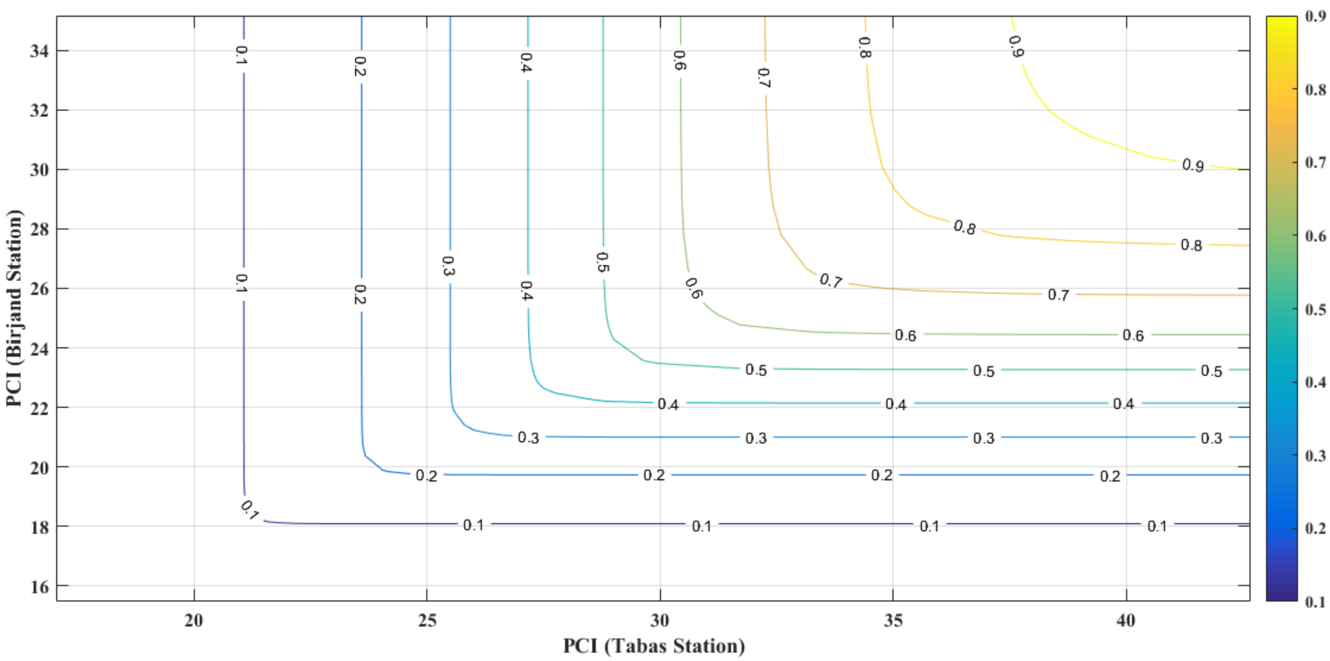

Fig. 5. Joint cumulative probability of PCI at Birjand and Tabas meteorological stations

According to Figure 4, the rainfall in each station can be estimated with different probabilities according to the rainfall at another station. For example, if the annual rainfall at Birjand meteorological station is equal to 240 mm, with a probability of 90%, the annual rainfall at Tabas meteorological station can be estimated about 140 mm. Also, for the annual rainfall equal 150 mm at Birjand meteorological station, with a probability of 40%, the annual rainfall at Tabas meteorological station can be estimated about 80 mm. Considering that these curves can be used as type curves for rainfall management and water harvesting if rainfall is predicted in a station, the rainfall at another station can be easily estimated with different probabilities that it is also true for Figure 5. According to Figure 5, it can be seen that the scattering and changes in the rainfall pattern at Tabas meteorological station, according to Table 2, are in a strong irregularity state. While the condition of Birjand meteorological station is better. According to Figures 4 and 5, it is easy to estimate the changes in the PCI at a station with different probabilities compared to another station. For example, if the PCI at Birjand meteorological station is 32, with a probability of 90%, the PCI at Tabas meteorological station is 38 for the same year. According to figure 5, different analyzes can be performed regarding the possibility of joint probability occurrence at the two studied stations. The changes in PCI at Tabas and Birjand meteorological stations during the statistical period were also presented in Figure 6. Change of PCI in Tabas station is more than birjand station because the number of days without rain in Tabas station is more than another one. Also, the distribution of rainfall between the months of the year in Tabas station is more irregular than Birjand station Which is consistent with the studies of Nazeri et al. (2016); Nazeri Tahroudi et al. (2019), Khozeymehnezhad et al. (2019).

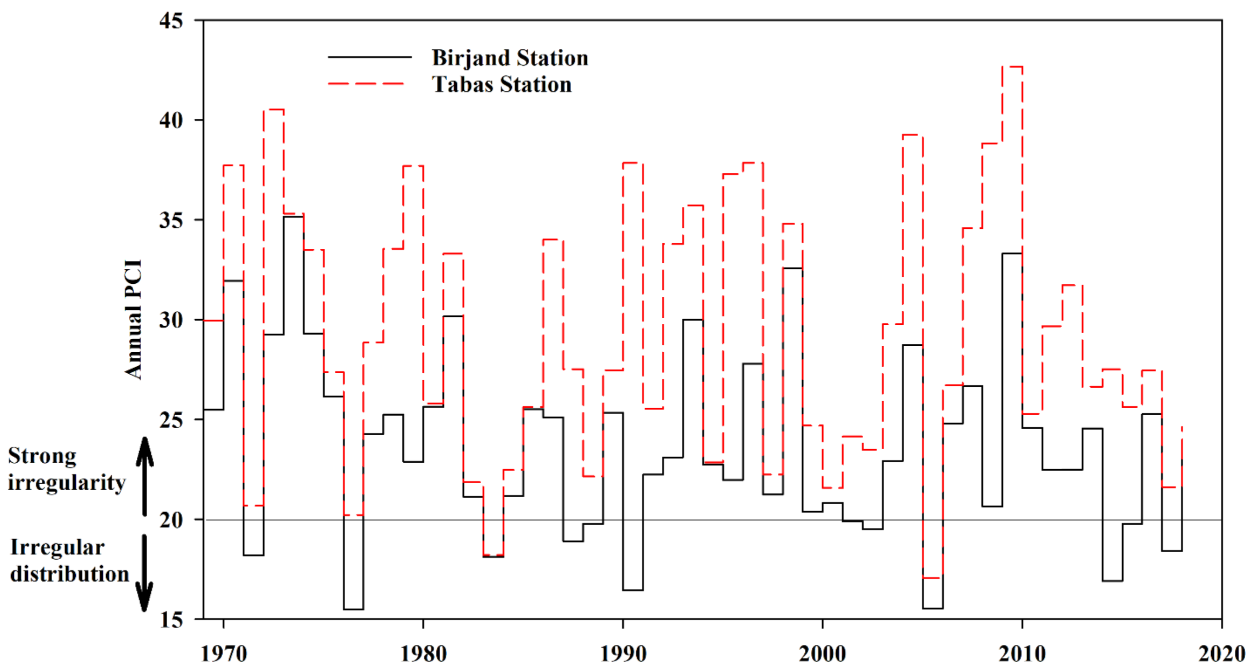

Fig. 6. Changes in PCI at Birjand and Tabas meteorological Stations in the period of 1969-2018

The results of PCI at the studied stations indicated that the PCI is irregular at the stations. The average PCI at Tabas and Birjand meteorological stations is about 23 and 29, respectively. Therefore, both of them are in a strong irregularity state based on Table 2. According to Figure 6, it can be seen that the changes in PCI during the statistical period have been increasing. This indicates an increase in irregularities in the distribution and pattern of rainfall. Increased irregularity in the precipitation concentration causes the rainfall to become extremely. This also indicates the irregular distribution of rainfall between the months of the year. According to the presented results, water storage and water harvesting in the studied region are extremely important.

4. Conclusion

The results of PCI at the studied stations showed that according to the PCI classification, the distribution of rainfall pattern at the studied stations were irregular (Birjand meteorological station) and strongly irregular (Tabas meteorological station), which has also increased in recent years. An increase in PCI will mean an increase in irregularities in rainfall distribution as well as an increase in extreme rainfalls and floods. Increasing the extreme rainfall will also increase the runoff, which will reduce the groundwater level and destroy the soil texture and its productivity. This makes the importance of water storage and water harvesting even more. Finally, the studied pair variables (rainfall-rainfall and PCI-PCI at Birjand and Tabas meteorological stations) while evaluating and confirming the existence of correlation and selecting appropriate marginal distribution functions, were fitted using different copula functions. The results of the evaluation statistics introduced Gumbel-Hougaard and Clayton copulas as the best copulas for the rainfall-rainfall and PCI-PCI pair variables at the studied stations, respectively. Joint cumulative probability curves were presented using selected copulas and their coefficients. The results showed that using the presented curves easily and with different probabilities, the rainfall and PCI of each station can be estimated with different probabilities based on another station. The use of copula functions in joint frequency analysis and conditional probability of occurrence is a new way in water resources management that considers the effective parameters. Simultaneous analysis of rainfall and PCI at two or more stations reveals the impact of this phenomenon. This method leads to the production of type curves to better water resources management in the region.

5. Conflicts of Interest

No potential conflict of interest was reported by the authors.

6. Acknowledgements

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